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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL REPORT
R-56

GRAPHICAL PRESENTATION OF DIFFERENCE SOLUTIONS FOR TRANSIENT RADIAL HEAT CONDUCTION IN HOLLOW CYLINDERS WITH HEAT TRANSFER AT THE INNER RADIUS AND FINITE SLABS WITH HEAT TRANSFER AT ONE BOUNDARY

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Cleveland, Ohio

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SUMMARY

Nondimensional temperature distributions for transient radial heat conduction through finite hollow cylinders and one-dimensional heat conduction in slabs of finite thickness are presented in graphical form for a range of heat input. The solutions are for radial heat conduction with heat transfer at the inner radius or slab heat conduction with heat transfer at one boundary. In both types of conduction it is assumed that the boundary opposite the heat-transfer surface is thermally insulated. The radial solutions cover a range of dimensionless radius ratios. The material is assumed to be homogeneous, and the physical properties are considered invariant with temperature. It is required that the heat-transfer coefficient and gas recovery temperature have quasi-steady-state values and the heat capacitance material temperature be essentially uniform at the start of heat addition. The solutions were obtained by means of difference equations and may be used for cooling as well as heating problems. These nondimensional solutions eliminate the need for obtaining solutions for each different material, heat-transfer coefficient, gas temperature, and initial material temperature. The range of variables should be adequate to cover rocket and missile calculations where a material is utilized as a heat capacitor.

INTRODUCTION

To the authors' knowledge, no solutions to the equation for transient radial heat conduction in a hollow cylinder with heat transfer at the inner radius and thermal insulation at the outer radius are available in the literature. Presumably this is due to the extreme labor involved in obtaining the closed-form solution in terms of Bessel functions. It was felt that these answers would be desirable for some types of rocket and missile heat-transfer calculations. Also, considering the accuracy of some of the assumptions often necessary in this field, it was felt that solutions of the difference equation would be sufficiently accurate for many engineering purposes. All of the results presented herein were obtained in a few days on a high-speed digital computer.

The case of one-dimensional heat conduction in a slab with heat transfer at one boundary and thermal insulation on the opposite boundary has been presented in the literature and may be found in graphical form in references 1 to 3. Solutions for this case were readily obtained on the computing machine by a slight modification of the difference equation for radial heat conduction and are presented as additions to the other information.

The range of variables covered should be adequate for most rocket and missile applications where a material may be used as a heat capacitor.

SYMBOLS

A	heat-flow area, sq ft
a	nondimensional parameter, $\Delta R^2/2\Delta\tau$
c	specific heat of material, Btu/(lb)(°R)
H	nondimensional heat-transfer parameter, hr_i/k
h	convective heat-transfer coefficient, Btu/(hr)(sq ft)(°R)
k	thermal coefficient of conductivity of material, Btu/(hr)(ft)(°R)
L	length of heat-flow path in slab, ft
M	factor for extending range of variables

m	number of $\Delta\tau$ increments
n	number of ΔR increments
q	heat-flow rate/unit area, Btu/(hr)(sq ft)
R	nondimensional radius, r/r_i
r	radius, ft
T	temperature, °R
t	time, hr
X	nondimensional distance, x/x_i
x, y, z	Cartesian coordinates, ft
α	coefficient of thermal diffusivity, $k/\rho c$, sq ft/hr
θ	circumferential measure, radians
ρ	material density, lb/cu ft
τ	nondimensional time, $\alpha t/r_i^2$ or $\alpha t/x_i^2$ as applicable
φ	nondimensional temperature ratio, $(T-T_0)/(T_g-T_0)$

Subscripts:

e	exterior (insulated surface) of material
g	gas or adiabatic wall
i	interior (heat-addition surface) of material
n	value of nondimensional temperature at $m=\tau/\Delta\tau$ time increments
n	value of nondimensional temperature at $n=(R-1)/\Delta R$ or $n=(X-1)/\Delta X$ distance increments from heat-addition surface
0	value at start of heating ($t=0$)
1	solution 1
2	solution 2

SOLUTION OF EQUATIONS

The general form of the transient heat-conduction equation with assumed constant material property values (ref. 4) and no heat generation may be written in vector notation as

$$\frac{k}{\rho c}\nabla^2 T=\frac{\partial T}{\partial t} \tag{1}$$

The final form of the equation depends upon the type of coordinate system selected that is most suitable to the physical situation.

SOLUTION OF RADIAL-HEAT-CONDUCTION EQUATION

The expression for equation (1) in cylindrical coordinate system is

$$\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{2}$$

where α is the coefficient of diffusivity ($k/\rho c$). In many applications, for example in rocket work, it may be sufficiently accurate to assume that heat flows only in the radial direction. It might also be a fair approximation to say that the material temperature is uniform at the start of heat addition and heat is added at the inner radius by a quasi-steady-state heat-transfer coefficient and driving temperature. In high-velocity flow the driving temperature is usually considered to be the adiabatic wall or recovery temperature, which is defined as the gas stream static temperature plus the product of the recovery factor and the dynamic temperature increase. With these assumptions, equation (2) simplifies to

$$\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{3}$$

with the boundary conditions,

$$q=-k\frac{\partial T}{\partial r}=h(T_g-T) \quad \text{at } r=r_i$$

$$\frac{\partial T}{\partial r}=0 \quad \text{at } r=r_e$$

$$T=T_0 \quad \text{at } t=0 \text{ for all } r$$

Equation (3) may be made nondimensional by making a change of variables with the following dimensionless parameters:

$$\left.\begin{aligned}\varphi&=\frac{T-T_0}{T_g-T_0}\\ \tau&=\frac{\alpha t}{r_i^2}\\ \text{and}\quad R&=\frac{r}{r_i}\end{aligned}\right\} \tag{4}$$

The resulting equation is

$$\frac{\partial\varphi}{\partial\tau}=\frac{\partial^2\varphi}{\partial R^2}+\frac{1}{R}\frac{\partial\varphi}{\partial R} \tag{5}$$

This equation can be considered as the formal limit, as the increments $\Delta R=\Delta r/r_i$ and $\Delta\tau=\alpha\Delta t/r_i^2$ tend to zero, of the difference equation

$$\frac{\varphi_{m+1,n}-\varphi_{m,n}}{\Delta\tau}=\frac{\varphi_{m,n+1}-2\varphi_{m,n}+\varphi_{m,n-1}}{\Delta R^2}+\frac{1}{R}\left[\frac{(\varphi_{m,n+1}-\varphi_{m,n})+(\varphi_{m,n}-\varphi_{m,n-1})}{2\Delta R}\right]$$

where m is the number of $\Delta\tau$ increments, and n is the number of ΔR increments. The difference equation may also be written as

$$\frac{\Delta R^2}{2\Delta\tau}(\varphi_{m+1,n}-\varphi_{m,n})=\frac{\varphi_{m,n+1}+\varphi_{m,n-1}}{2}-\varphi_{m,n}+\frac{\Delta R}{4R}(\varphi_{m,n+1}-\varphi_{m,n-1}) \quad (6)$$

If $a=\Delta R^2/2\Delta\tau$ and it is realized that $R=R_i+n\Delta R=1+n\Delta R$, the solution for φ at $\tau=(m+1)\Delta\tau$ in terms of φ at $\tau=m\Delta\tau$ is

$$\varphi_{m+1,n}=\frac{\varphi_{m,n+1}+\varphi_{m,n-1}}{2a}+\left(1-\frac{1}{a}\right)\varphi_{m,n}+\frac{\Delta R}{4a(1+n\Delta R)}(\varphi_{m,n+1}-\varphi_{m,n-1}) \quad (7)$$

where n ranges from 1 to $n_e-1=\frac{(r_e/r_i)-1}{\Delta R}-1=\frac{R_e-1}{\Delta R}-1.$

The inner-radius boundary condition relevant to the difference-equation formulation would be of the form

$$h(T_g-T_{m,0})=-k\left(\frac{T_{m,1}-T_{m,0}}{\Delta r}\right)$$

Using the nondimensional parameters of equation (4) together with the additional nondimensional heat-transfer parameter, $H=hr_i/k$, this equation may be expressed as

$$\varphi_{m,0}=\frac{H\Delta R+\varphi_{m,1}}{1+H\Delta R} \quad (8)$$

The material is assumed to be insulated at the exterior radius r_e, and it may be assumed that the heat that reaches the last increment of material serves to raise the temperature of this increment. Mathematically, this may be stated as

$$-kA\,\frac{\partial T}{\partial r}=\rho cA\Delta r\,\frac{\partial T}{\partial t}\ \text{or}\ -\frac{\alpha}{\Delta r}\,\frac{\partial T}{\partial r}=\frac{\partial T}{\partial t}$$

Using the nondimensional parameters, the exterior-radius boundary condition relevant to the difference-equation formulation would be of the form

$$\varphi_{m+1,n_e}=\left(1-\frac{1}{2a}\right)\varphi_{m,n_e}+\frac{1}{2a}\,\varphi_{m,n_e-1} \quad (9)$$

where $R_e=r_e/r_i=1+n_e\Delta R$ or $n_e=(R_e-1)/\Delta R$ and $a=\Delta R^2/2\Delta\tau$.

The solution for the transient radial-heat-conduction problem can be obtained by use of equations (7), (8), and (9) for any selected values of nondimensional heat-transfer parameter H and exterior to interior radius ratio $R_e=r_e/r_i$.

Calculations were made on a high-speed digital computer using the IPIAC coding system. This system makes use of two-word floating point arithmetic of over ten significant figures calculation accuracy with an average multiplication or addition taking 1 millisecond. The interrupt feature of the computer saves time on output of results, so that a case with 30 increments in R and 5000 steps of integration required 25 minutes to get 1100 numbers of output. Without the interrupt feature, the punching of paper tape would have required another 5 minutes. The longest case computed for this presentation (80 increments in R, 10,000 steps in τ) required approximately 3 hours of calculation.

SOLUTION OF SLAB-HEAT-CONDUCTION EQUATION

In Cartesian coordinate system, equation (1) can be expressed as

$$\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

If the heat is assumed to flow only in the x-direction perpendicular to the heat-transfer surface in a rectangular slab with all other surfaces thermally insulated, this equation and the appropriate boundary conditions may be stated as

$$\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2} \quad (10)$$

$$q=-k\frac{\partial T}{\partial x}=h(T_g-T) \qquad \text{at } x=x_i$$

$$\frac{\partial T}{\partial x}=0 \qquad \text{at } x=x_e$$

$$T=T_0 \qquad \text{at } t=0 \text{ for all } x$$

where x is measured from an arbitrary reference point, x_i units from the heat-transfer surface.

Equation (10) may be nondimensionalized by changing variables with the following parameters:

$$\varphi=\frac{T-T_0}{T_g-T_0}$$

$$\tau=\frac{\alpha t}{x_i^2}$$

and

$$X=\frac{x}{x_i}$$

with the resulting equation

$$\frac{\partial\varphi}{\partial\tau}=\frac{\partial^2\varphi}{\partial X^2}$$

This equation can be considered as the formal limit, as the increments $\Delta X=x/x_i$ and $\Delta\tau=\alpha\Delta t/x_i^2$ tend to zero, of the difference equation,

$$\varphi_{m+1,n}=\frac{\varphi_{m,n+1}+\varphi_{m,n-1}}{2a}+\left(1-\frac{1}{a}\right)\varphi_{m,n} \qquad (11)$$

where $a=\Delta X^2/2\Delta\tau$. Equation (11) is identical to equation (7) with the last term on the right omitted. Again, the values of n in equation (11) range from 1 to $n_e-1=\frac{(x_e/x_i)-1}{\Delta X}-1=\frac{X_e-1}{\Delta X}-1$. The boundary conditions at x_i and x_e are of identical form as in the radial-heat-conduction case. Therefore, appropriate nondimensionalized difference equations of the boundary conditions are

$$\varphi_{m,0}=\frac{H\Delta X+\varphi_{m,1}}{1+H\Delta X} \qquad (12)$$

and

$$\varphi_{m+1,n_e}=\left(1-\frac{1}{2a}\right)\varphi_{m,n_e}+\frac{1}{2a}\varphi_{m,n_e-1} \qquad (13)$$

where $X_e=x_e/x_i=1+n_e\Delta X$ or $n_e=(X_e-1)/\Delta X$.

The solution for the transient slab-heat-conduction problem can be obtained by use of equations (11), (12), and (13). It may be noted that equation (11) differs from equations (6) and (7) in that X does not appear in the solution which permits the use of x_i as an arbitrary dimension providing it is used in the determination of τ, X, and X_e.

After solutions were obtained for the slab-heat-flow problem, the results were converted to more conventional parameters by the following equations:

$$\frac{\alpha t}{L^2}=\frac{\tau}{(X_e-1)^2}$$

$$\frac{x}{L}=\frac{X-1}{X_e-1}$$

$$\frac{hL}{k}=H(X_e-1)$$

where x is the distance in the direction of heat flow from the heated surface in a slab of length L, and where the temperature is computed at time t.

Results for the slab-heat-conduction calculations were obtainable by omitting the last term from equation (7) in the machine program for the radial-heat-flow solution.

RANGE OF VARIABLES

A value of $a=1.25=\Delta R^2/2\Delta\tau$ was used for all calculations. Reference 5 indicates that, for the slab case, $a\geq 1$ is required to guarantee stability of the solution. Seven values of the nondimensional heat-transfer coefficient H were used for the radial-heat-conduction calculations. They were 0.2, 0.5, 1.1, 2, 5, 10, and 20. Eight values of the outside to the inside radius ratio $R_e=r_e/r_i$ were used, but not all were used with every heat-transfer value. The following table summarizes the cases computed. The value listed is the upper limit of the nondimensional time τ computed:

R_e	H						
	0.2	0.5	1.1	2	5	10	20
	τ						
1.1	---	---	---	---	0.1	0.05	0.05
1.2	---	---	---	---	.1	.05	.05
1.3	---	---	---	---	.1	.05	.05
1.4	5	5	0.1	0.1	---	----	----
2.0	5	5	5	5	---	----	----
2.5	5	5	5	5	---	----	----
3.0	5	5	5	5	---	----	----
4.0	5	5	5	5	---	----	----

A study of the range of variables needed indicated that the ones used would be adequate to cover most rocket and missile work where a material is utilized as a heat capacitor. The nondimensional temperature parameter φ rises very rapidly when small values of R_e are used along with high values of the heat-transfer parameter H. Therefore, only a small range of τ need be computed before φ becomes very close to 1.0. Since the region of interest is at the lower τ for these cases, the computations were stopped at the values indicated by the table.

Cases that ran to an upper limit of τ equal to 5 used a ΔR interval of 0.05. To keep the computational details comparable, cases that ran to upper limits of $\tau=0.1$ and 0.05 used a ΔR interval equal to 0.005.

Applications of these computations to large rockets indicated that the heat-transfer parameter H would be high and the value of the outside to inside radius R_e would be low or close to 1.0. If equation (5) is examined for values of R_e close to unity, one sees that $1/R$ may be considered approximately equal to 1.0 for all values of R. Consequently, the results of the case for $R_e=1.1$ could be extended to new cases by using the following multiplying factor M:

$$H_2=MH_1$$

$$\tau_2=\tau_1/M^2$$

$$R_{e,2}=1+\frac{R_{e,1}-1}{M}$$

For minimum error, the extension should not be attempted for cases where R_e is higher than 1.1. A check of the accuracy was made by extending the computed values for $H_1=10$ and $R_{e,1}=1.2$ to $H_2=20$ and $R_{e,2}=1.1$ (i.e., $M=2$) and comparing this with the calculated results for the second case. This comparison is illustrated in figure 1. The extension equations were considered to be accurate enough that no solutions were carried out for H greater than 20 and R_e less than 1.1.

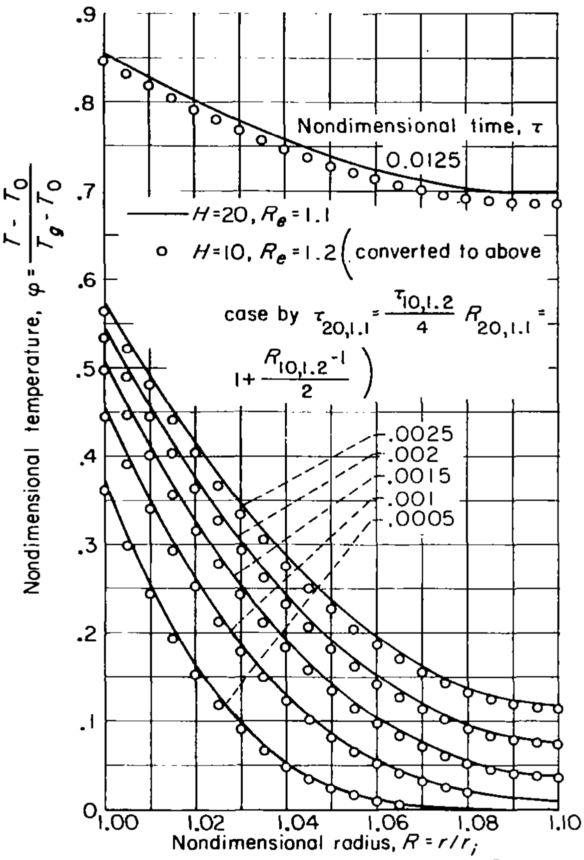

FIGURE 1.—Comparison of solution using multiplying factors and solution using difference equations.

RESULTS

Whenever a solution to a problem is obtained by using a finite-difference equation, the accuracy of the solution depends upon the size of the increments used and how accurately the approximations for the derivatives represent the true case. As a qualitative check of the radial-heat-conduction results, comparisons were made with closed-form solutions presented in reference 6. These solutions were made for the infinite material case; therefore, the comparisons hold only for the period of time when the outer boundary has experienced no change in temperature.

Figure 2 shows the comparison between the closed-form solution for infinite material and the solution as obtained from equations (7), (8), and (9) for a nondimensional heat-transfer parameter H equal to 0.2 and an outside to inside radius ratio R_e equal to 4.0. The solution should duplicate the closed-form solution up to the nondimensional time τ equal to 0.6, because the outer boundary ($R=4.0$) has experienced very little change in temperature up to this value of τ and thus corresponds to the case for infinite material.

Since the results from reference 6 are given in graphical form with a very rough grid and also because the two cases are not quite comparable, no quantitative judgment as to accuracy will be made. The trends at this early time, for which the solutions are least accurate, indicate that the answers are sufficiently good for most engineering purposes. Reference 7 gives the nondimensional temperature distribution for the one-dimensional heat-conduction case in a finite material in the form of an infinite series. Using the first six

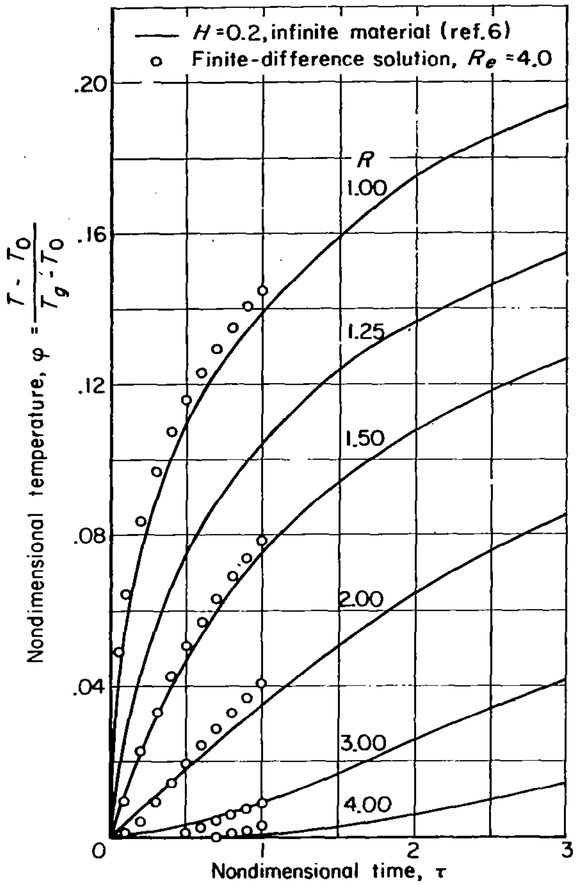

FIGURE 2.—Comparison of closed-form solution for infinite material and solution by difference equations for radial heat flow.

terms of this series for $x/L=0.2$ and for the range of $0.2 \leqq hL/k \leqq 6$, the results from the finite-difference equations agree within approximately 5 percent at a value of $\alpha t/L^2=0.017$, within approximately 1 percent at a value of $\alpha t/L^2=0.5$, and within approximately 0.2 percent at $\alpha t/L^2=1.0$. Since, in many cases, temperatures of the gas and heat-transfer coefficients cannot be determined with accuracies greater than this, the increments selected were considered to be small enough. Also, material properties may vary appreciably with temperature, so that any solutions to the linear equations are in themselves approximations. Further reduction of the material and time increments causes the machine time for calculations to be greatly increased and poses machine storage problems.

RADIAL HEAT CONDUCTION

Plots of the nondimensional temperature φ against the nondimensional time τ for values of radius ratio R from 1.0 to R_e are presented in figure 3 for all the cases indicated in the preceding table.

SLAB HEAT CONDUCTION

The calculations for the slab-heat-conduction problems were carried out for the cases in the preceding table, where the upper limit of τ was equal to 5.0, with X_e replacing R_e in the table.

Plots are presented in two ways for the slab-heat-conduction solutions. Figure 4 presents plots of φ against $\alpha t/L^2$ from $x/L=0$ to 1.0 for values of hL/k equal to 0.2, 0.5, 1.1, 2.0, 4.0, and 6.0. Figure 5 presents plots of φ against hL/k for $\alpha t/L^2$ from 0.0125 to 1.2 for values of x/L equal to 0, 0.2, 0.4, 0.6, 0.8, and 1.0.

SOME APPLICATIONS FOR THESE SOLUTIONS

Figure 4 gives the time-temperature relations for any point in a slab of material (where the heat flow is only in one direction) for various values of the nondimensional heat-transfer parameter hL/k. For example, if an insulated slab or constant-diameter rod of known material and initial temperature T_0 has heat applied to the uninsulated end, and the time for the other end of the material to reach a certain temperature is required, this type of plot would be very useful. Estimates for the heat-transfer coefficient h and the adiabatic wall temperature T_g would first be made. Then, knowing the material properties (α and k) against temperature, the nondimensional parameters φ and hL/k could be computed. Entering the correct plot (hL/k) with the value φ and $x/L=1.0$, one can read a value of $\alpha t/L^2$ and finally compute a value of time t.

The graphs may be used for solving cooling as well as heating problems.

The plots of φ against hL/k for various values of x/L (fig. 5) can be used to find the heat-transfer coefficient h. For example, a constant-diameter rod of known material can be imbedded in a rocket nozzle in such a way that heat flow is along the axis of the rod with insulated sides and outer end. A thermocouple can be used to give the time-temperature relation for a given position in the rod for a rocket firing run. The adiabatic wall temperature T_g can be computed from rocket performance, and the initial temperature of the rod T_0 is known from the thermocouple readings before firing, enabling calculation of the nondimensional temperature φ. Then, knowing the material properties against temperature and the time of the thermocouple reading, $\alpha t/L^2$ can be computed. Entering the correct plot (x/L known) with values of φ and $\alpha t/L^2$, one can read hL/k. Again, knowing the length (L) and material property (k), one can compute the heat-transfer coefficient h.

The properties of materials (k, ρ, and c) may vary appreciably with temperature, and the problem arises concerning which temperature in the material should be used in the evaluation. Reference 8 shows that the material properties for simple metals for this type of computation should be evaluated at a temperature that is about one-fourth of the maximum if the initial temperature is zero, or at $\left(\frac{T_{x/L=0}-T_0}{4}+T_0\right)$ if the initial temperature is nonzero. This may be due to the fact that the temperature for evaluating properties should be averaged for time and distance in the material. The factor 4 appears to be a rough approximation for this value.

An iteration process might be needed to get the required accuracy if estimates used to start the problem are found to be quite inaccurate. As an illustration in the second example, the temperature of the rod at $x/L=0$ is needed to estimate the material properties α and k. This value could be approximated; and, once a value of hL/k is obtained, one could enter the plot of φ against hL/k for $x/L=0$, using the hL/k and $\alpha t/L^2$ computed, and see if the value of φ at $x/L=0$ gives a temperature close to the original estimate. If not, the process could be repeated until the desired convergence is obtained.

In the design of a rocket nozzle where a material is to be utilized as a heat capacitor, the appropriate radial heat-conduction solution (fig. 3) would be applicable at each axial station. To give the reader some feel for the nondimensional parameters used, two cases will be computed.

In the following table are the assumptions used and the calculated values arrived at by use of figures 3(a)(3) and 3(e)(4) for the throat of two rockets of different size. It was assumed that the wall material was graphite and it was necessary to find the firing duration, which is limited by the time required for the interior surface at the throat to reach 3830° R.

	Case I	Case II
Thrust, lb	600, 000	6000
Adiabatic wall temperature, T_s, °R	6030	6030
Initial graphite temperature, T_0, °R	530	530
Interior limiting temperature, T, °R	3830	3830
Rocket chamber pressure, lb/sq in. abs	300	300
Thrust coefficient	1. 347	1. 347
Rocket throat radius, r_i, in.	21. 74	2. 17
Exterior graphite radius at throat, r_e, in.	23. 91	4. 34
Material thickness, r_e-r_i, in.	2. 17	2. 17
Heat-transfer coefficient, h, Btu/(hr)(sq ft)(°R)	638	638
Graphite coefficient of diffusivity, α, sq ft/hr	1. 18	1. 18
Graphite thermal conductivity, k, Btu/(hr)(ft)(°R)	57. 8	57. 8
H	20	2
R_e	1. 1	2
R (for limiting temperature)	1. 0	1. 0
φ (limiting value)	0. 6	0. 6
τ	0. 003	0. 45
Firing duration, t, sec	30	40. 5

No comparisons between cases I and II should be drawn, because the local heat-transfer coefficient h is a function of the rocket radius, and these cases were given only to illustrate the nondimensional parameters.

By means of cross-plotting, curves of φ against H with τ as a parameter for fixed values of R and R_e can be made similar to the slab case for evaluating h from experimental data from a circumferentially and axially segmented rocket nozzle or chamber.

CONCLUDING REMARKS

Nondimensional-temperature distributions for transient radial heat conduction through hollow cylinders and one-dimensional heat conduction in slabs of finite thickness have been presented in graphical form for a range of heat input. The solutions are for radial heat flow with heat transfer at the inner radius or slab heat flow with heat transfer at one boundary. In both types of conduction it was assumed that the boundary opposite the heat-transfer surface is thermally insulated. The radial solutions cover a range of dimensionless radius ratios. The material is assumed to be homogeneous, and the physical properties are considered invariable with temperature. The solutions were obtained by means of difference equations, and calculations were made on a high-speed digital computer.

Some examples of uses for the curves might be as follows for any type of material:

1. The temperature distribution through a slab or hollow cylinder can be determined for known heat-transfer coefficient and adiabatic wall temperature.
2. Heat-transfer coefficients can be obtained from temperature measurements made at known times.
3. Heat-addition duration may be determined for given material thicknesses, heat-transfer coefficients, and limiting inner-wall temperatures.
4. Material thicknesses required for limiting wall temperatures and heat-addition duration may be obtained for known heat-transfer coefficients. This might be used in a weight analysis where various materials are used.
5. The graphs may be used for solving cooling as well as heating problems.
6. Thermal stresses arising during heating or cooling could be obtained from the temperature distribution through the material.
7. The solutions might be applicable to any diffusion process other than heat diffusion that satisfies the type of boundary conditions used herein.

Restrictions upon the use of these solutions are that there be no heat flow at any boundary except at the heat input surface, and the heat-transfer coefficient and driving temperature have quasi-steady-state values. The heat-capacitance material temperature must be essentially uniform at the start of heat addition.

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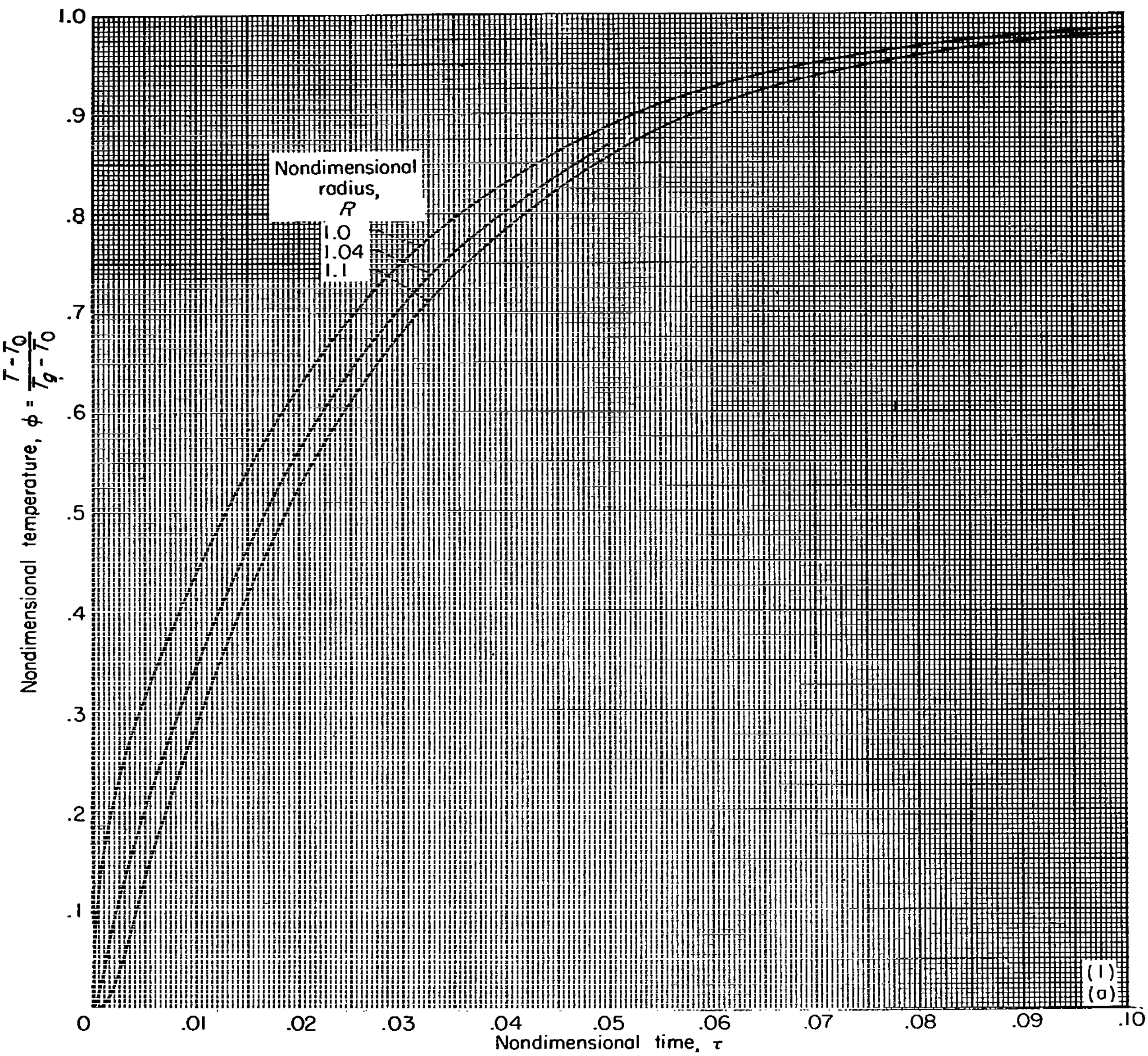
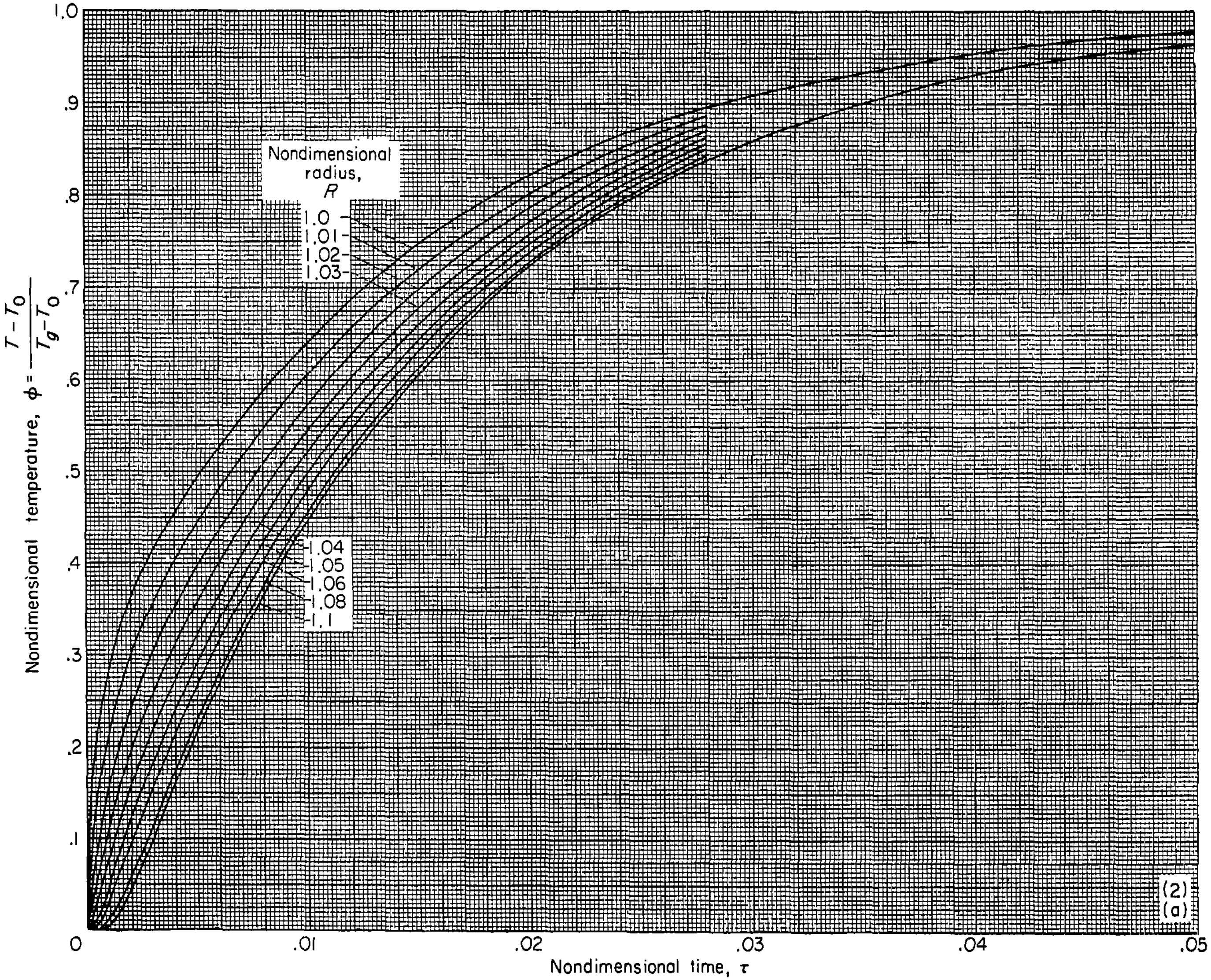


(1) $H=5.0$.
(a) $R_e=1.1$.

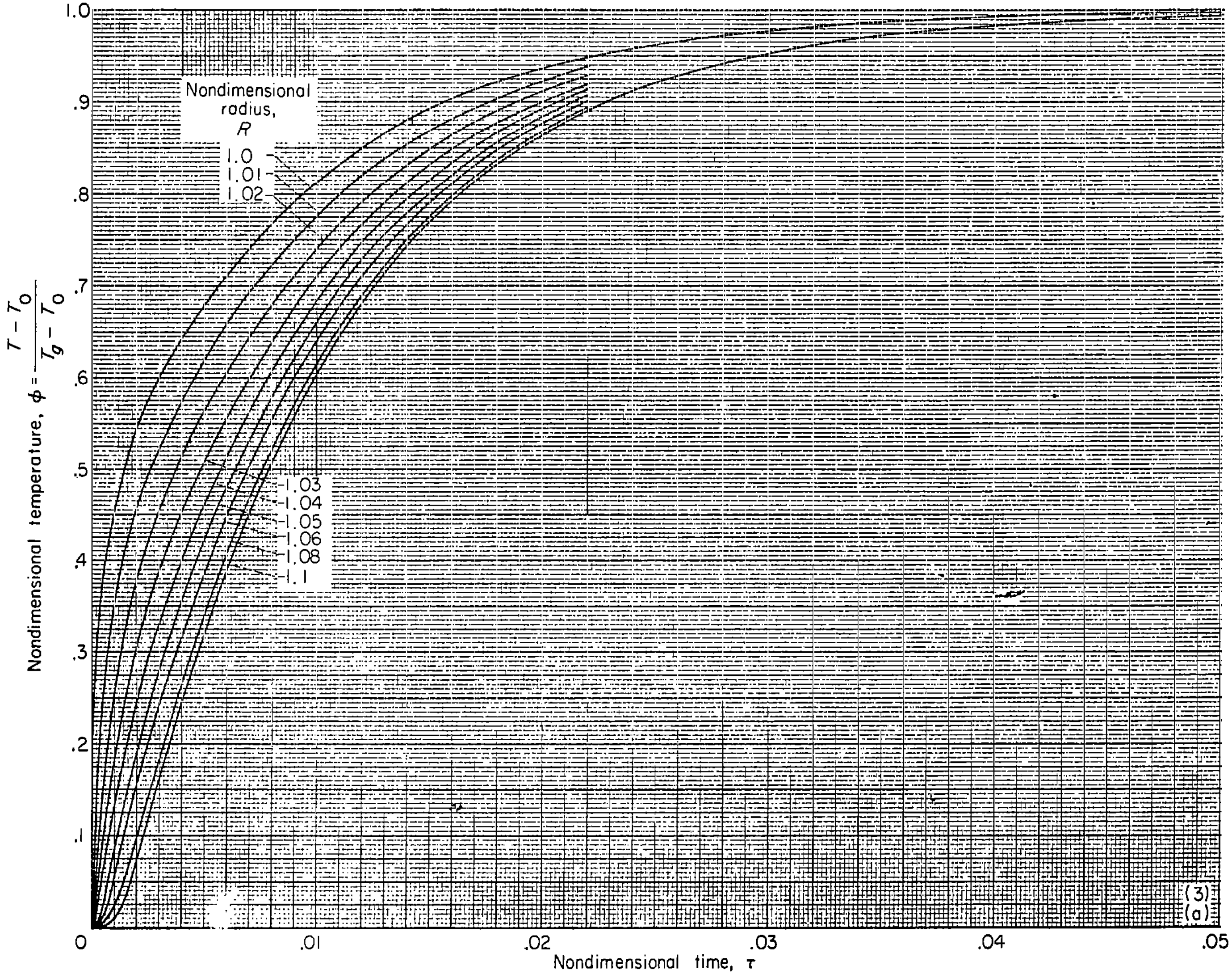
FIGURE 3.—Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder (Larger copies of all parts of figure 3 are available from NASA, Washington 25, D.C.)



(2) $H=10.0$.

(a) Continued. $R_e=1.1$.

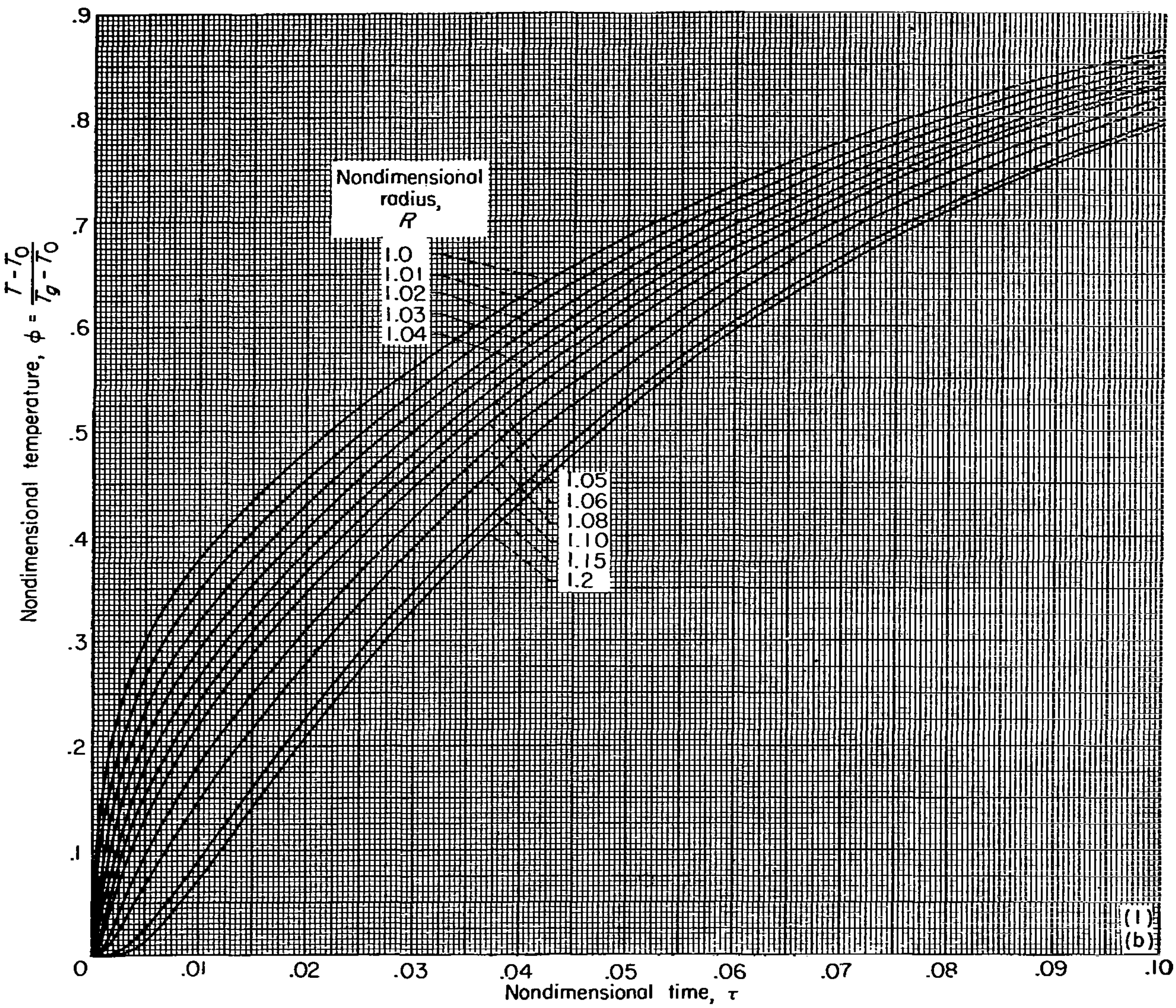
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) $H=20.0$.

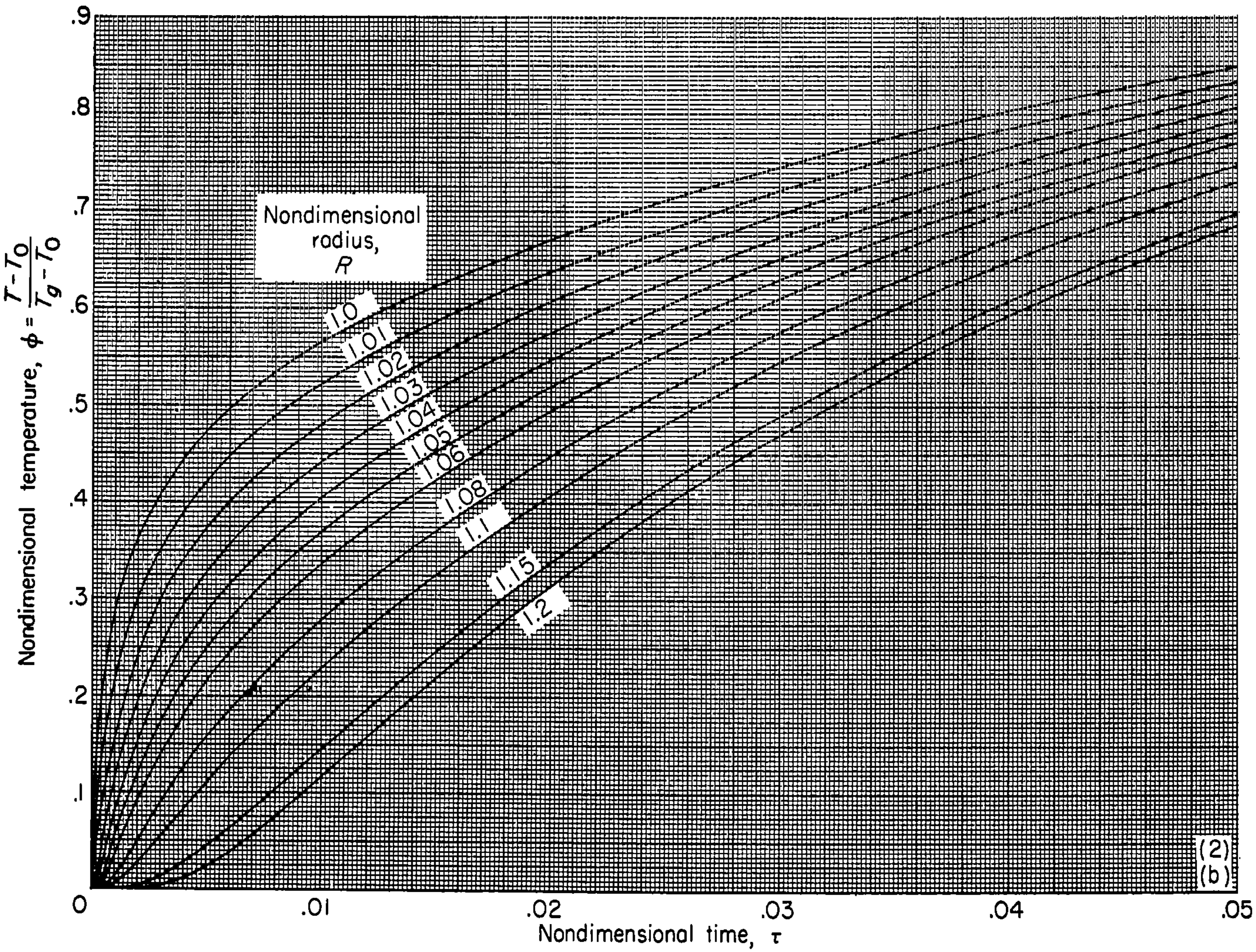
(a) Concluded. $R_e=1.1$.

Figure 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(1) $H=5.0$.
(b) $R_e=1.2$.

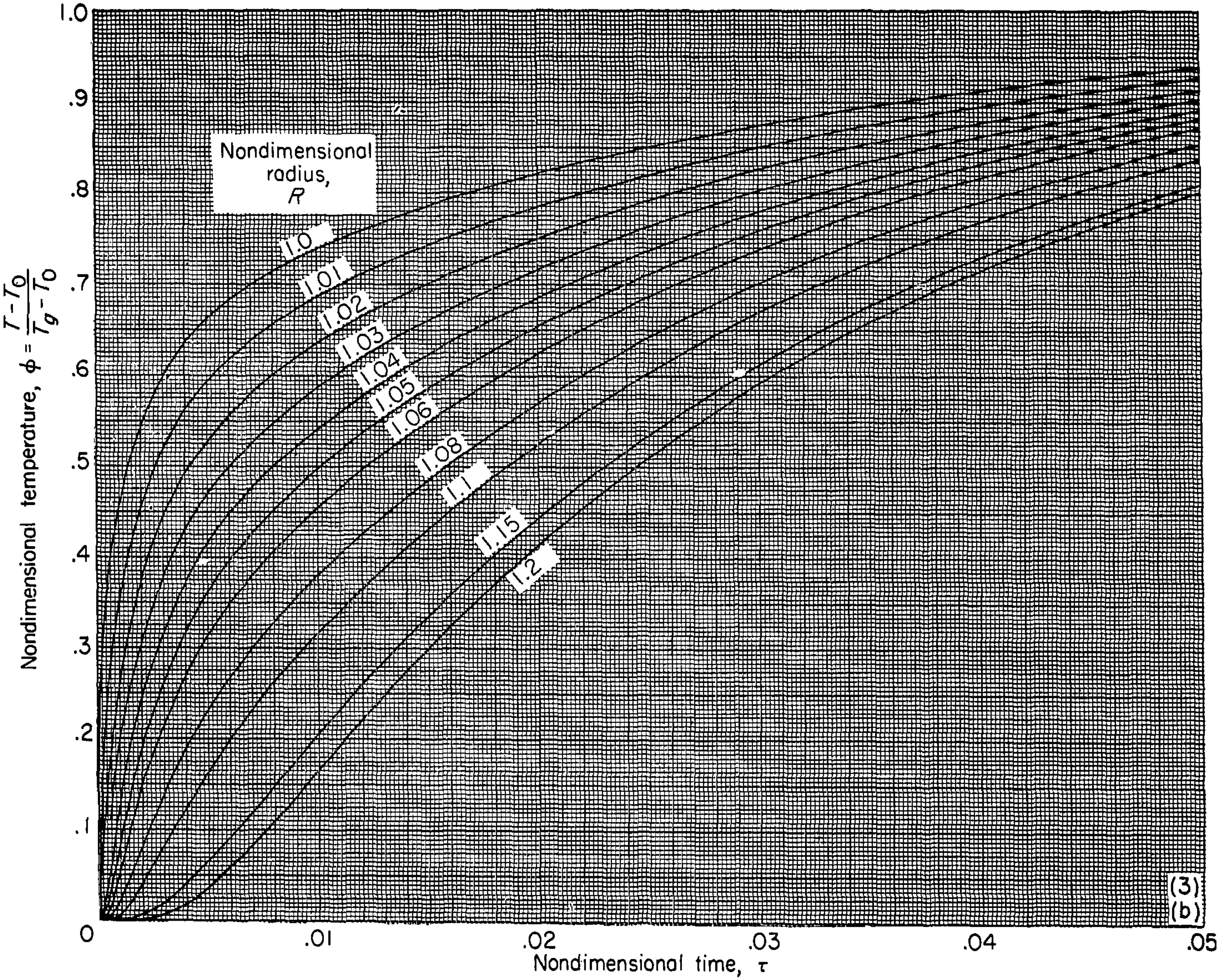
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(2) $H=10.0$.

(b) Continued. $R_e=1.2$.

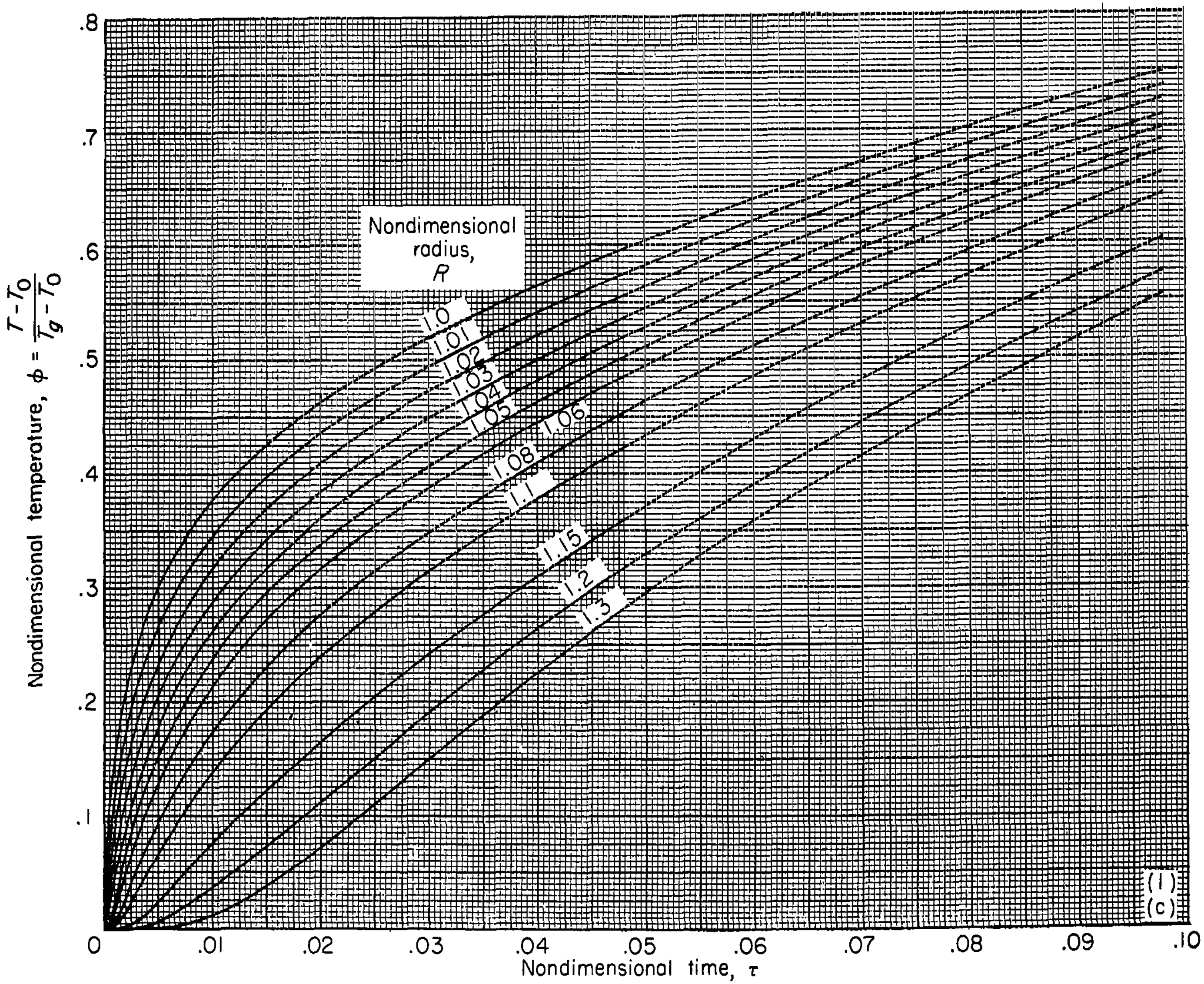
Figure 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) $H=20.0$.

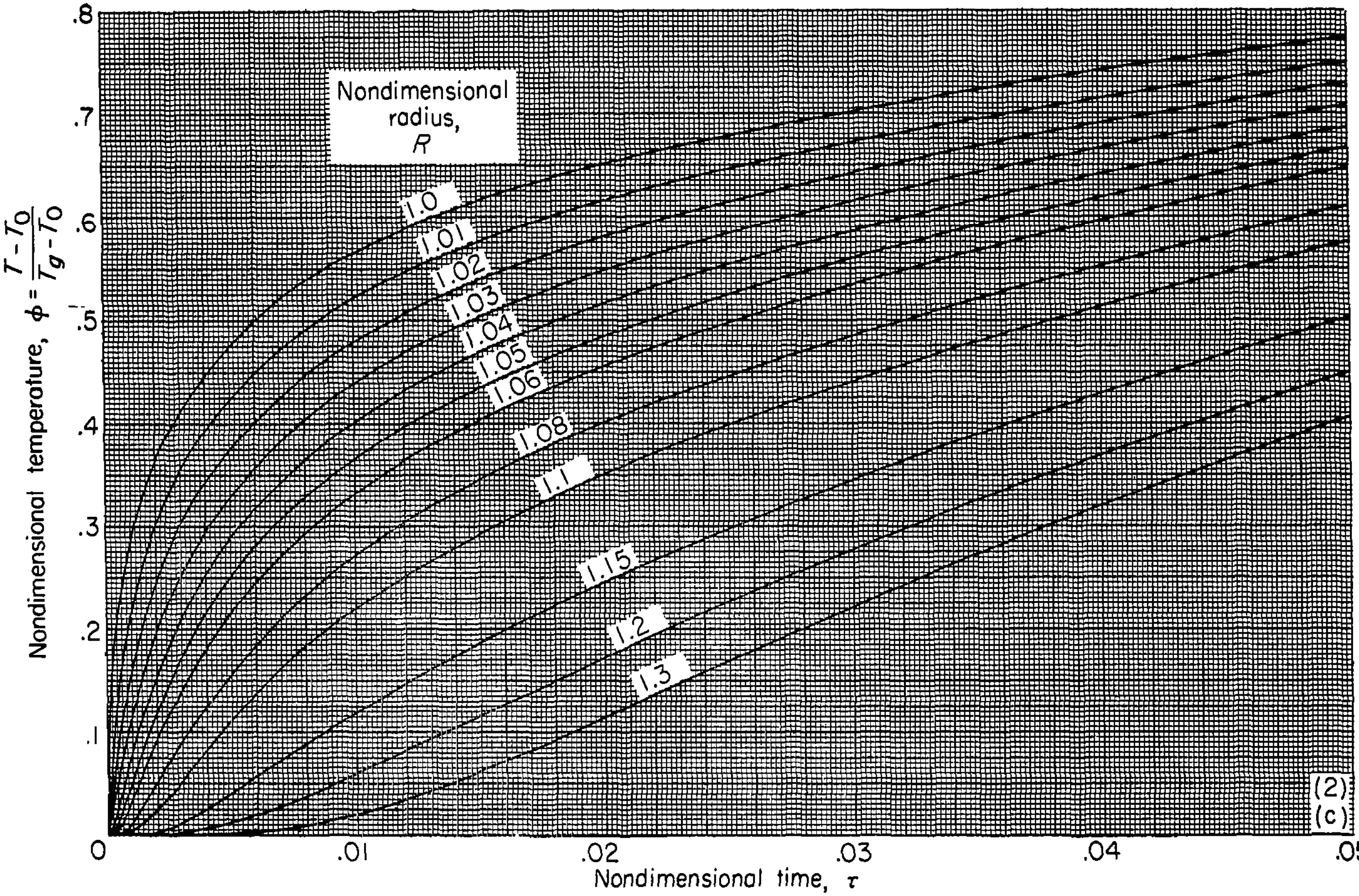
(b) Concluded. $R_e=1.2$.

FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



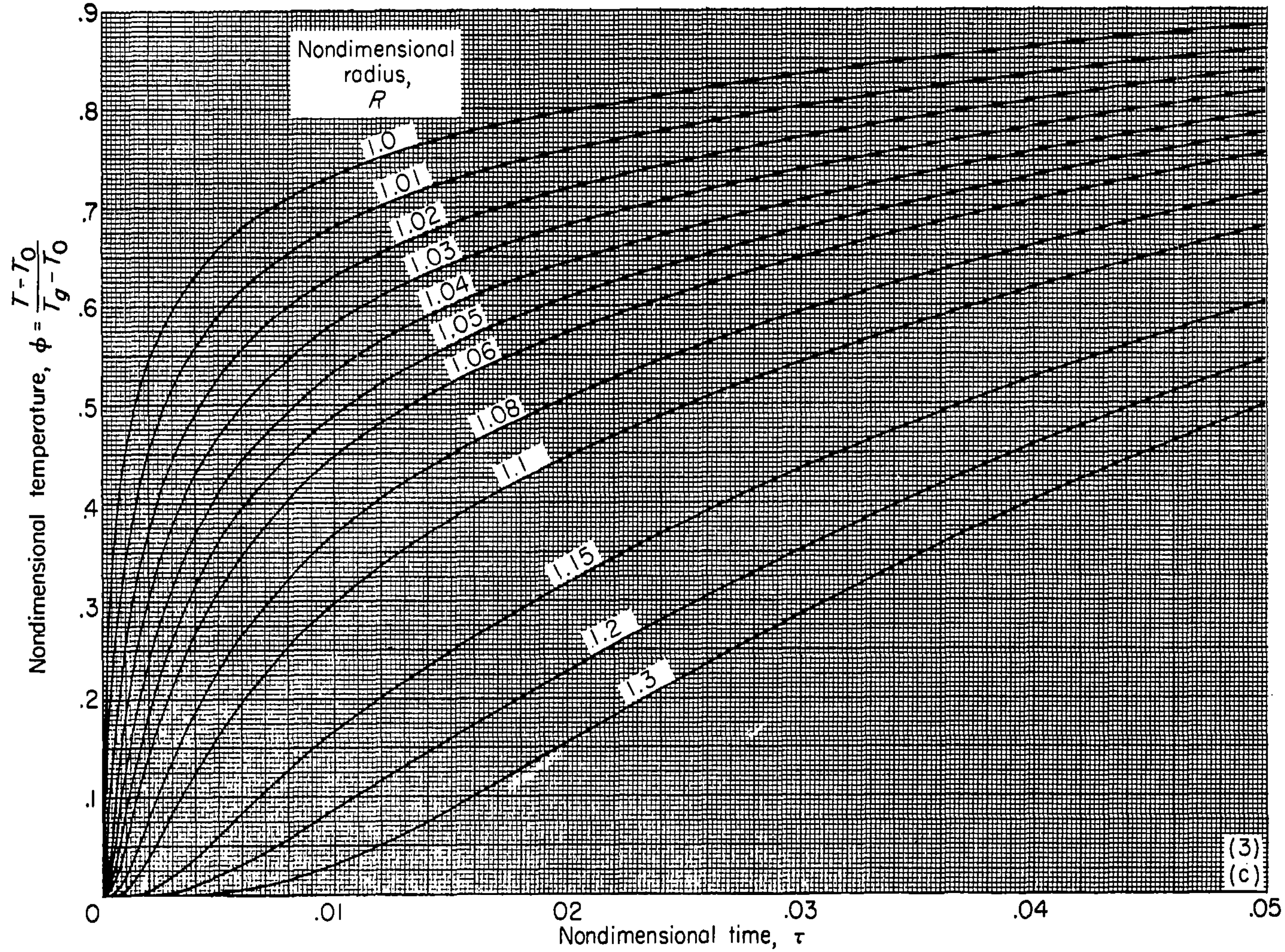
(1) $H=5.0$.
(c) $R_o=1.3$.

FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_o and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(2) $H=10.0$.
(c) Continued. $R_e=1.3$.

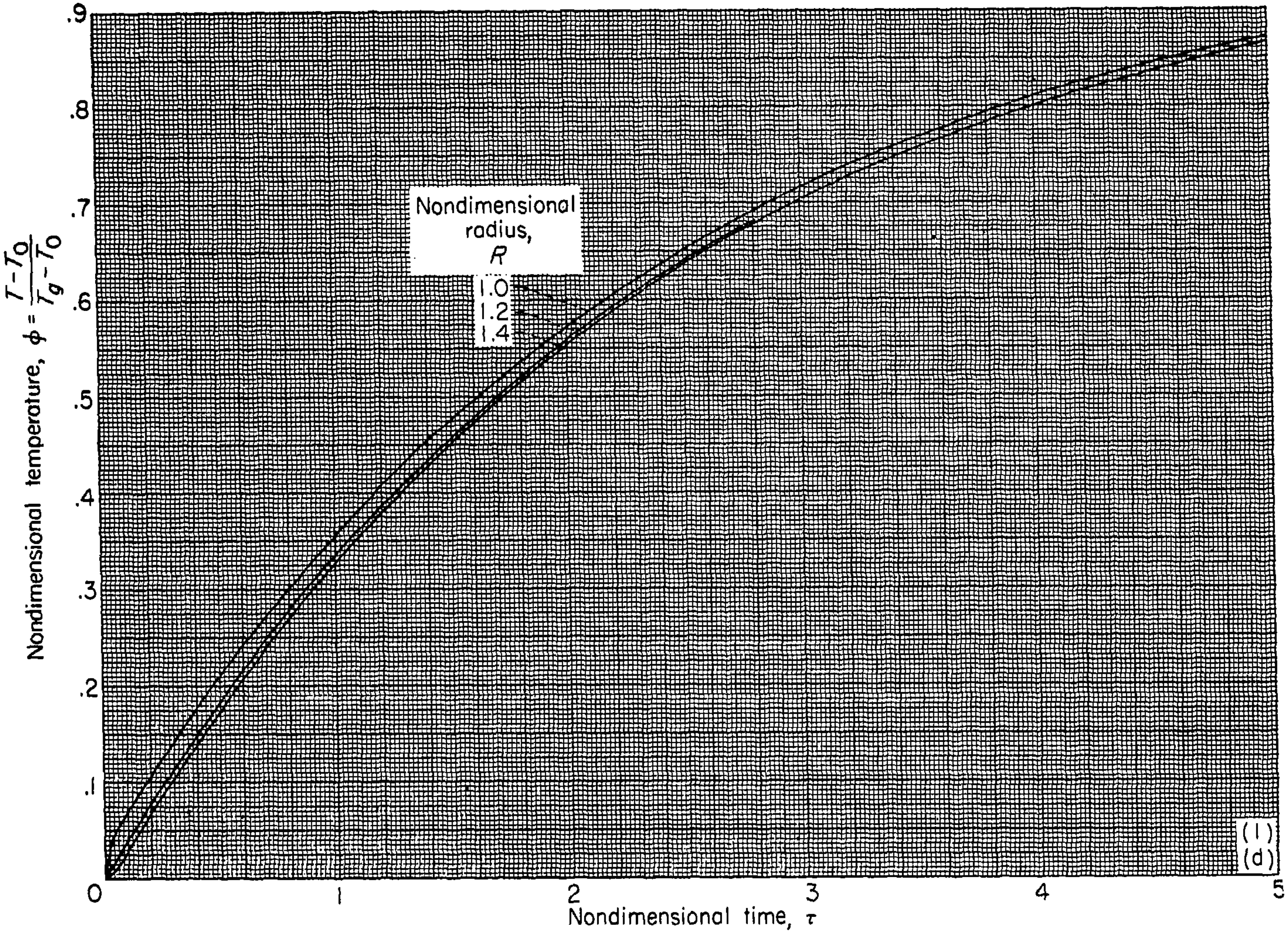
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) $H=20.0$.

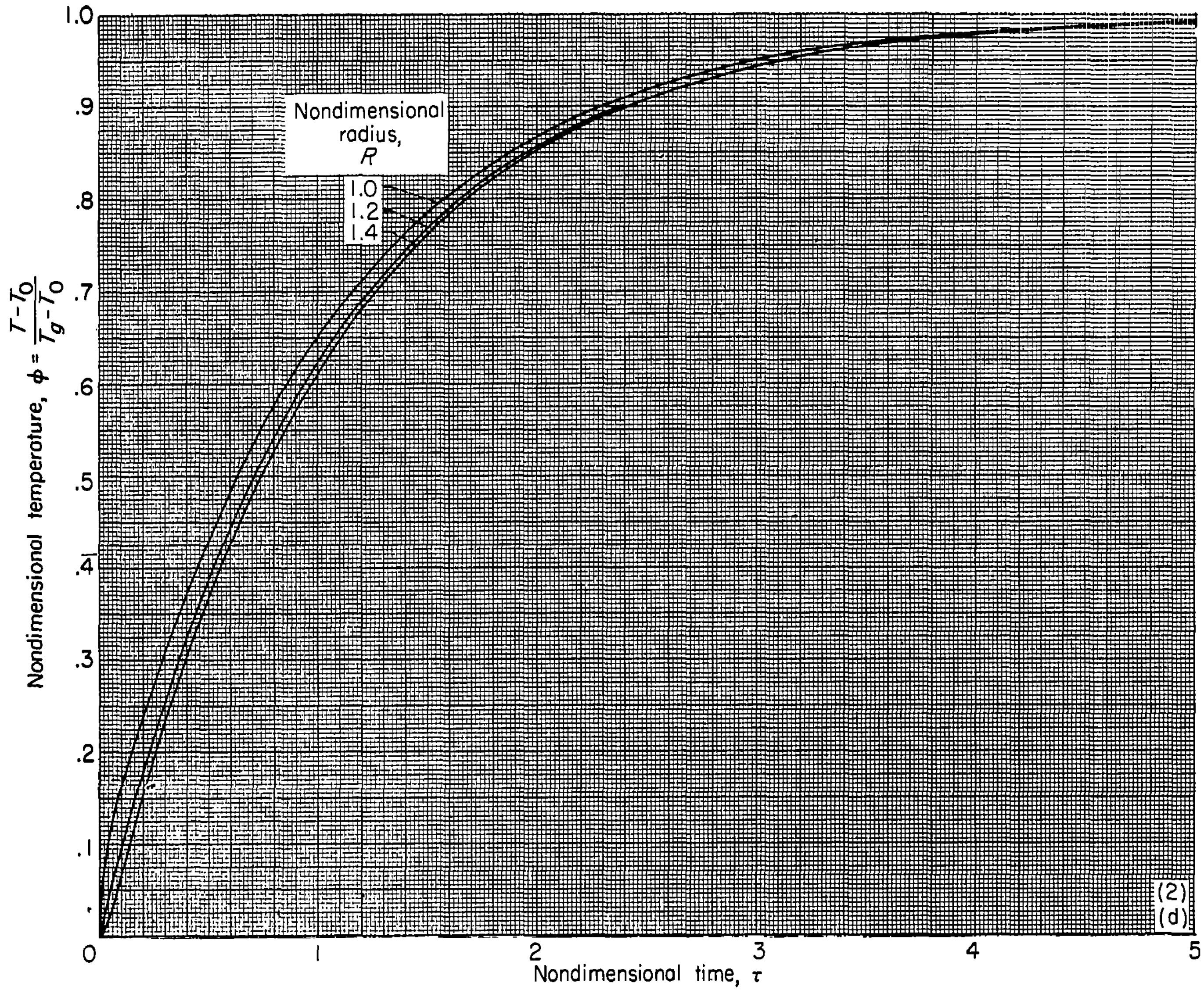
(c) Concluded. $R_e=1.3$.

FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(1) $H=0.2$.
(d) $R_s=1.4$.

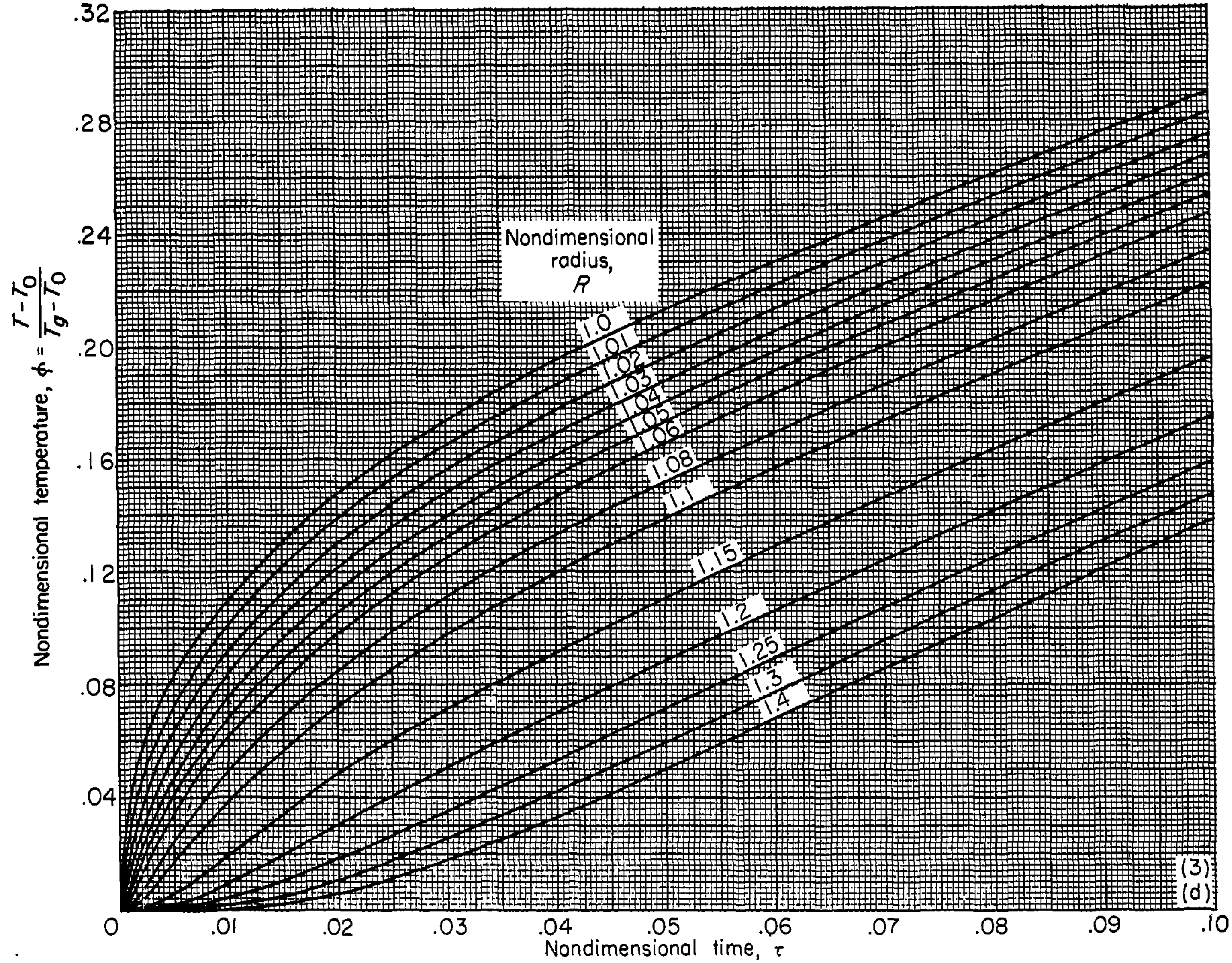
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_s and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(2) $H=0.5$.

(d) Continued. $R_e=1.4$.

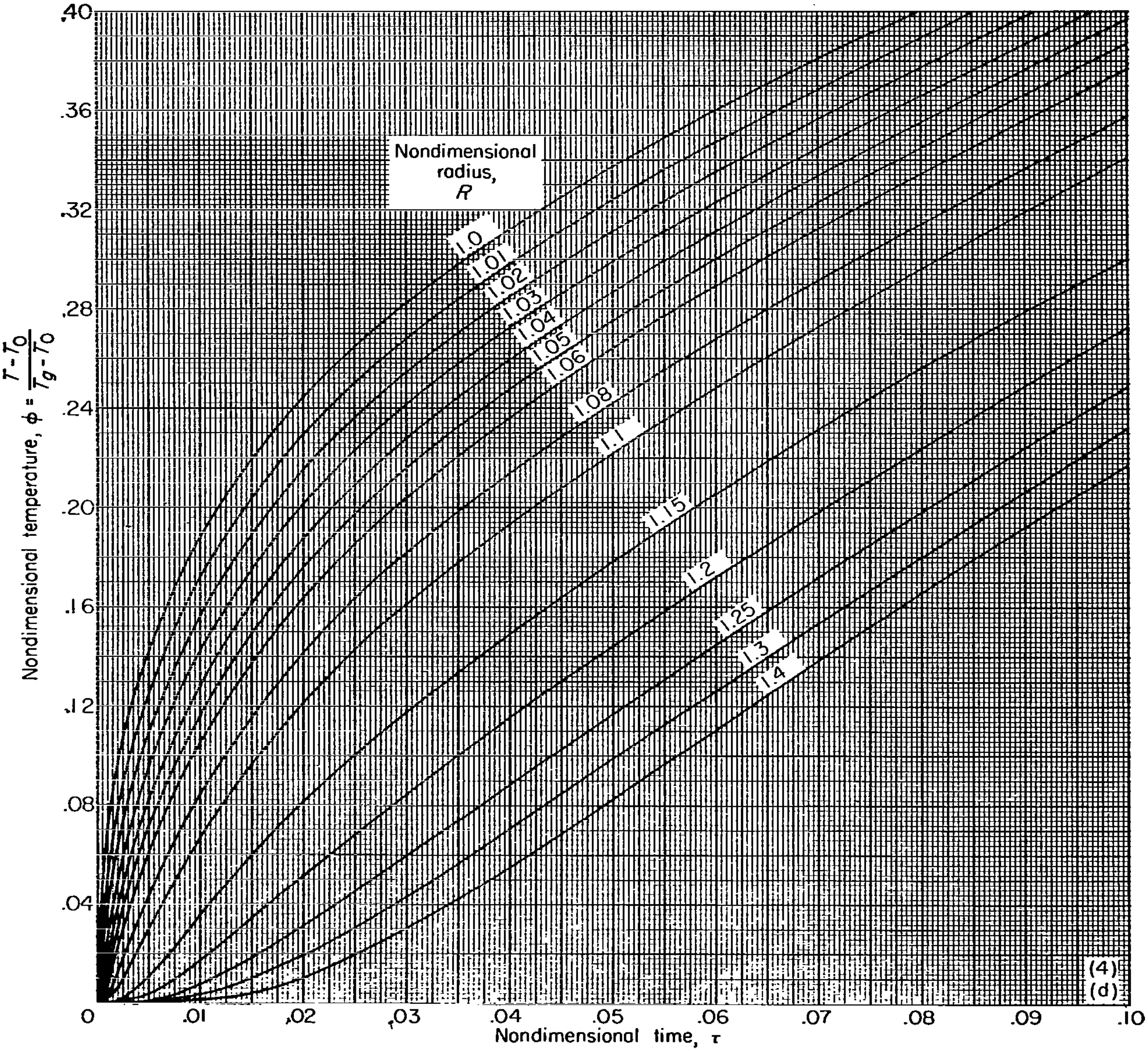
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) $H=1.1$.

(d) Continued. $R_e=1.4$.

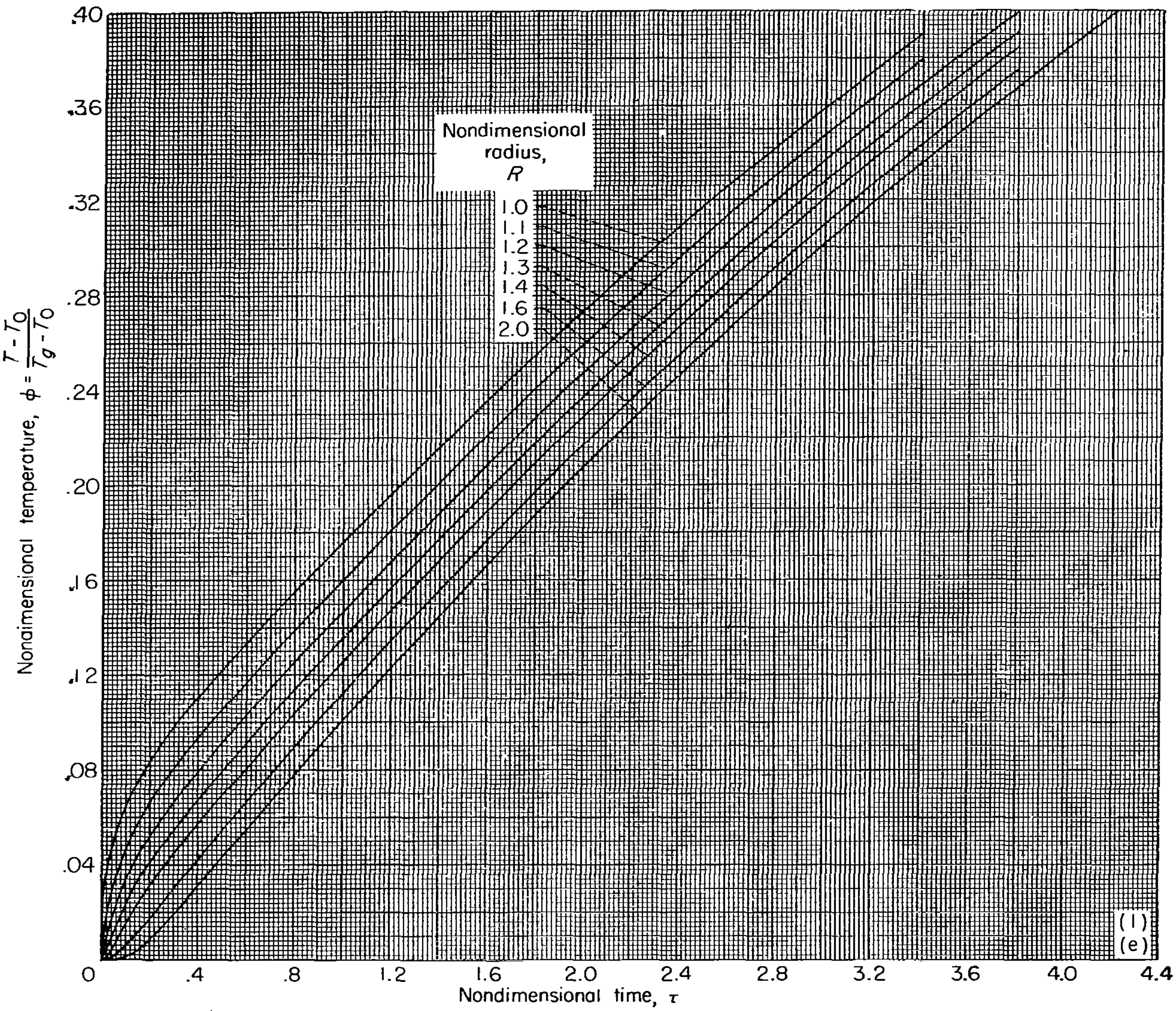
Figure 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(4) $H=2.0$.

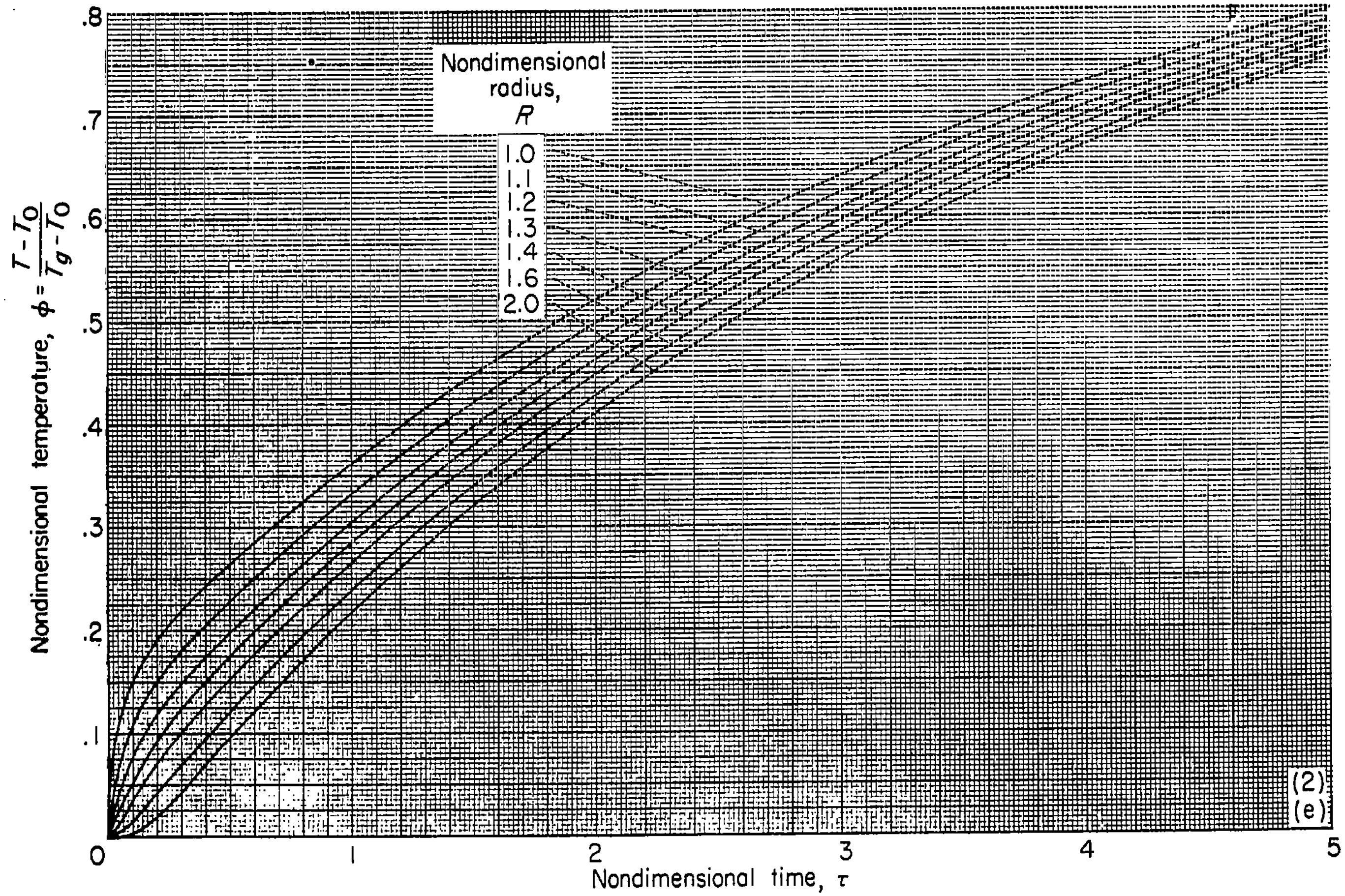
(d) Concluded. $R_e=1.4$.

FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(l) $H=0.2$.
(e) $R_e=2.0$.

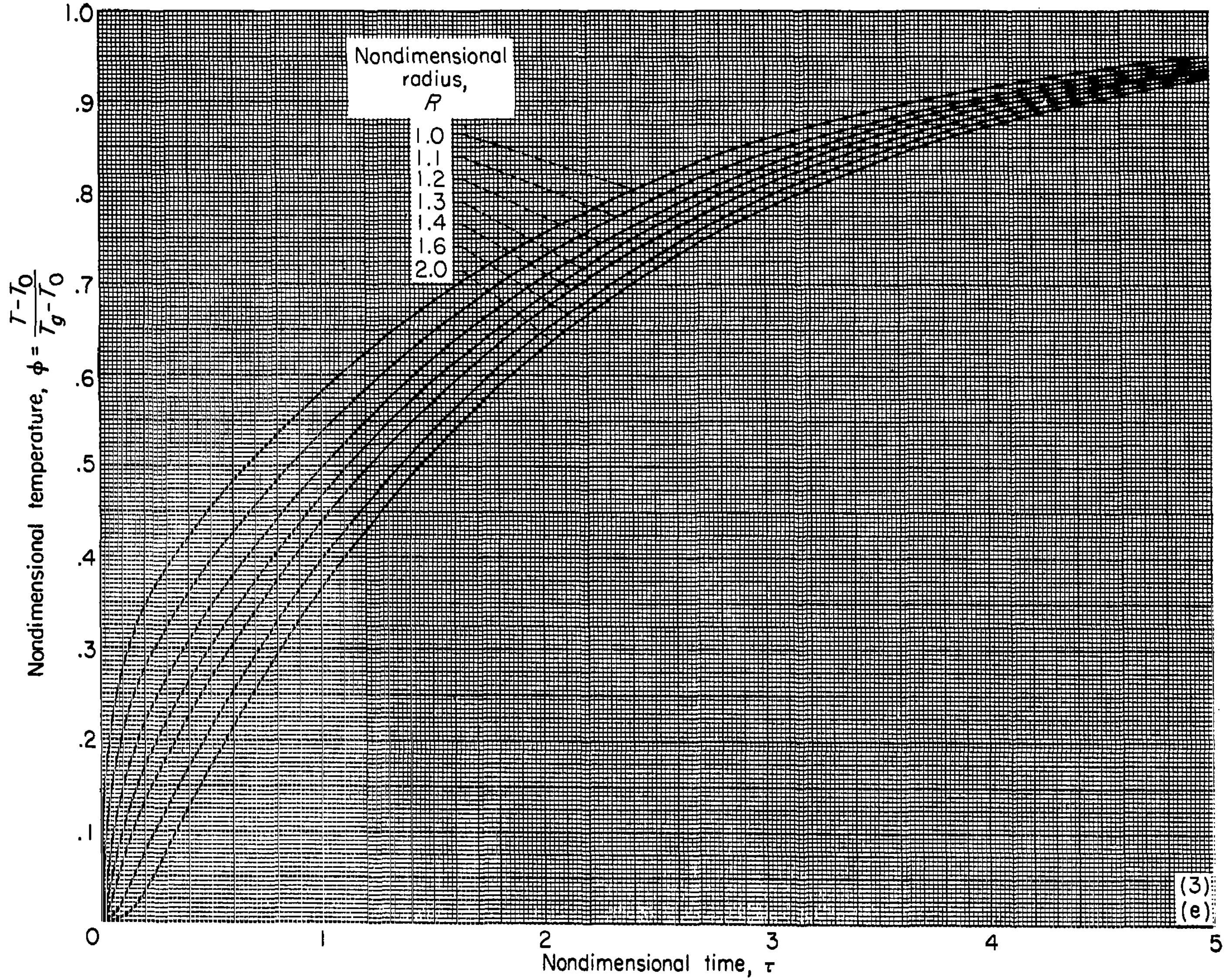
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(2) $H=0.5$.

(e) Continued. $R_e=2.0$.

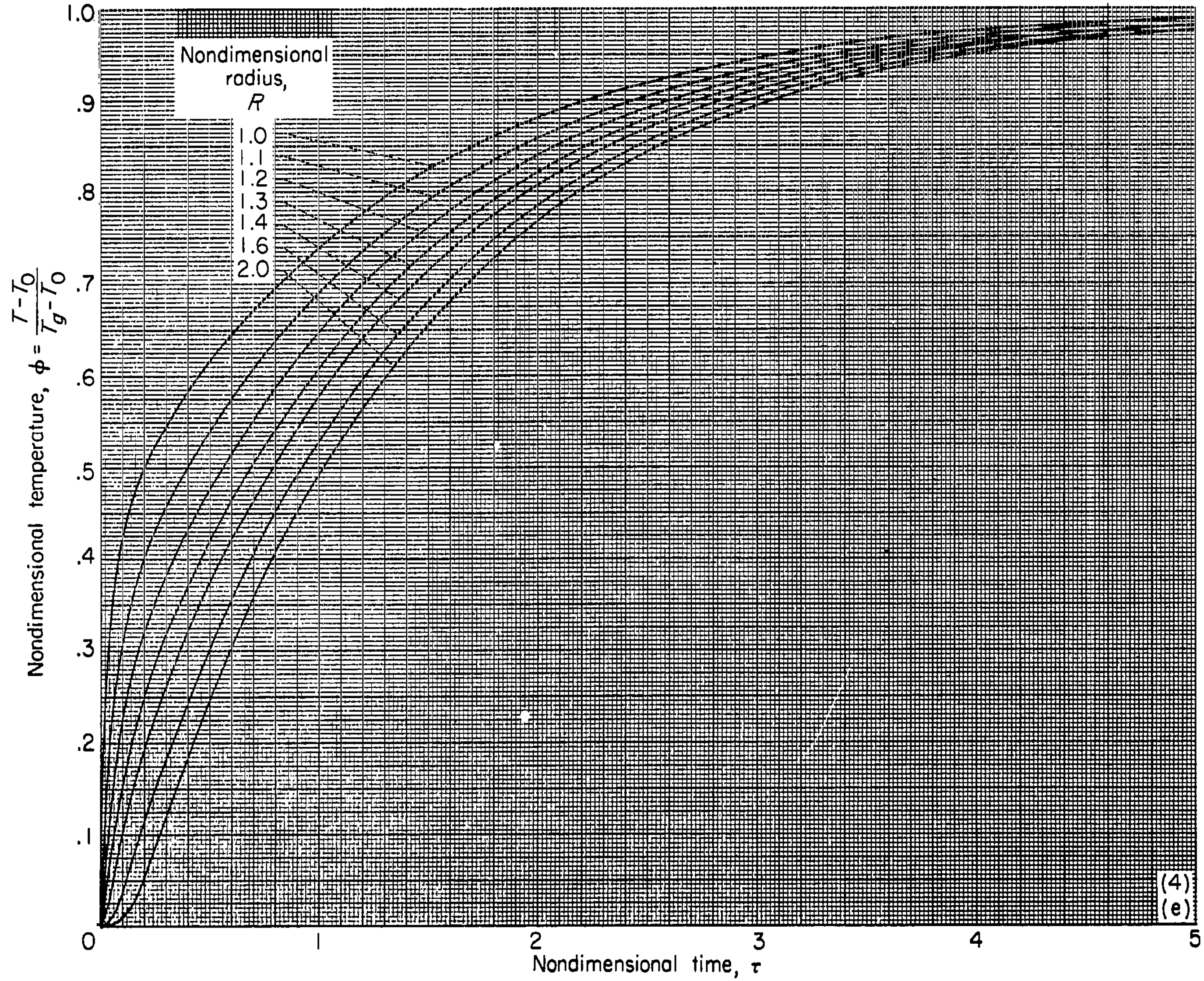
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) $H=1.1$.

(e) Continued. $R_e=2.0$.

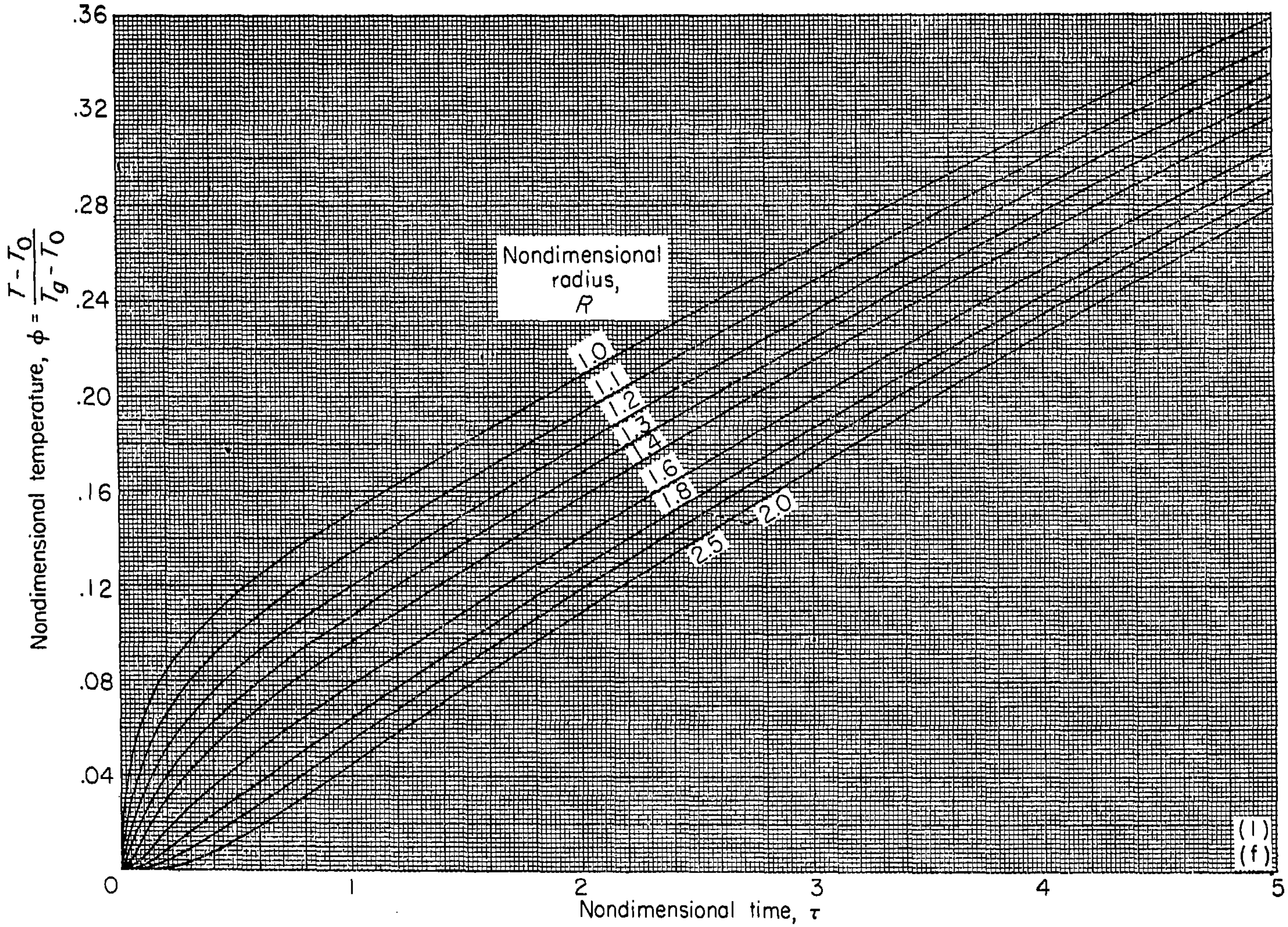
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(4) $H=2.0$.

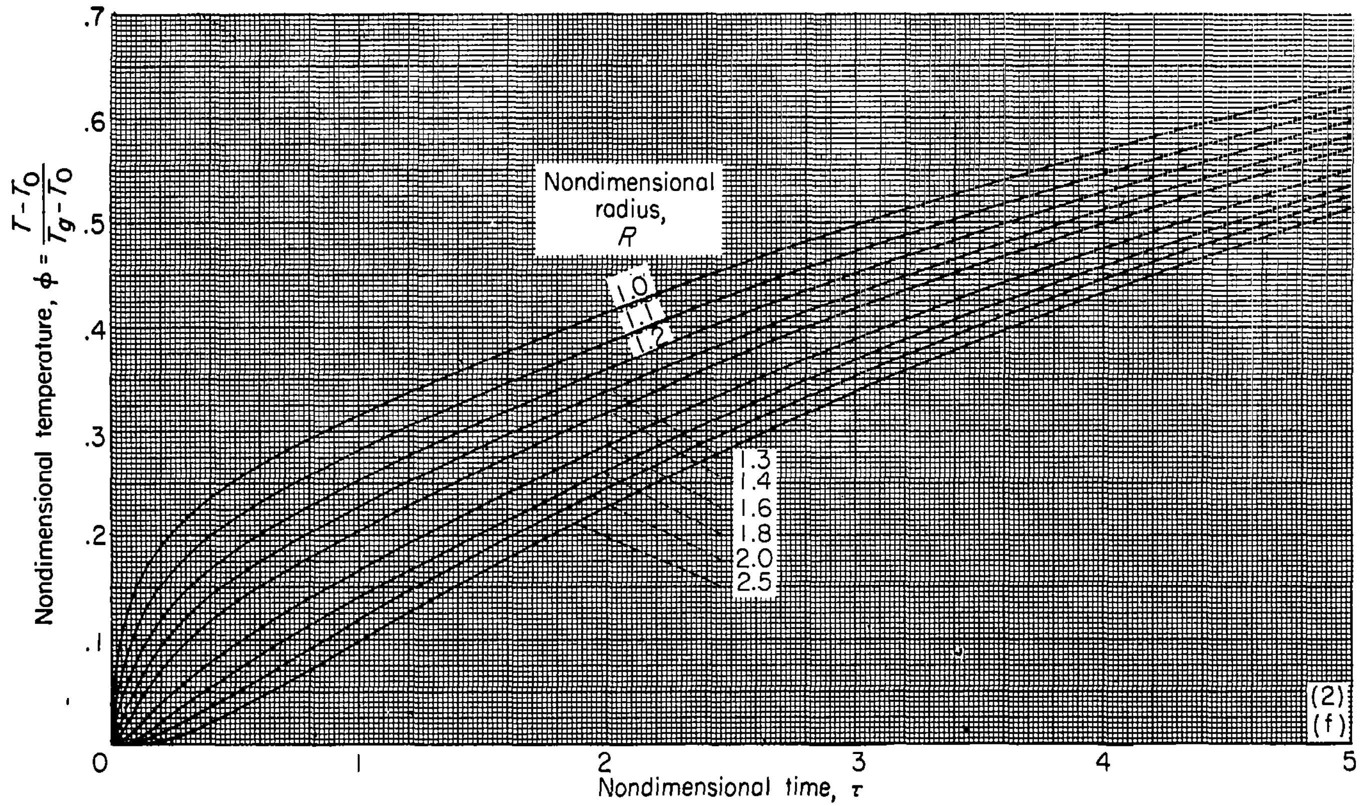
(e) Concluded. $R_e=2.0$.

FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(l) $H=0.2$.
(f) $R_e=2.5$.

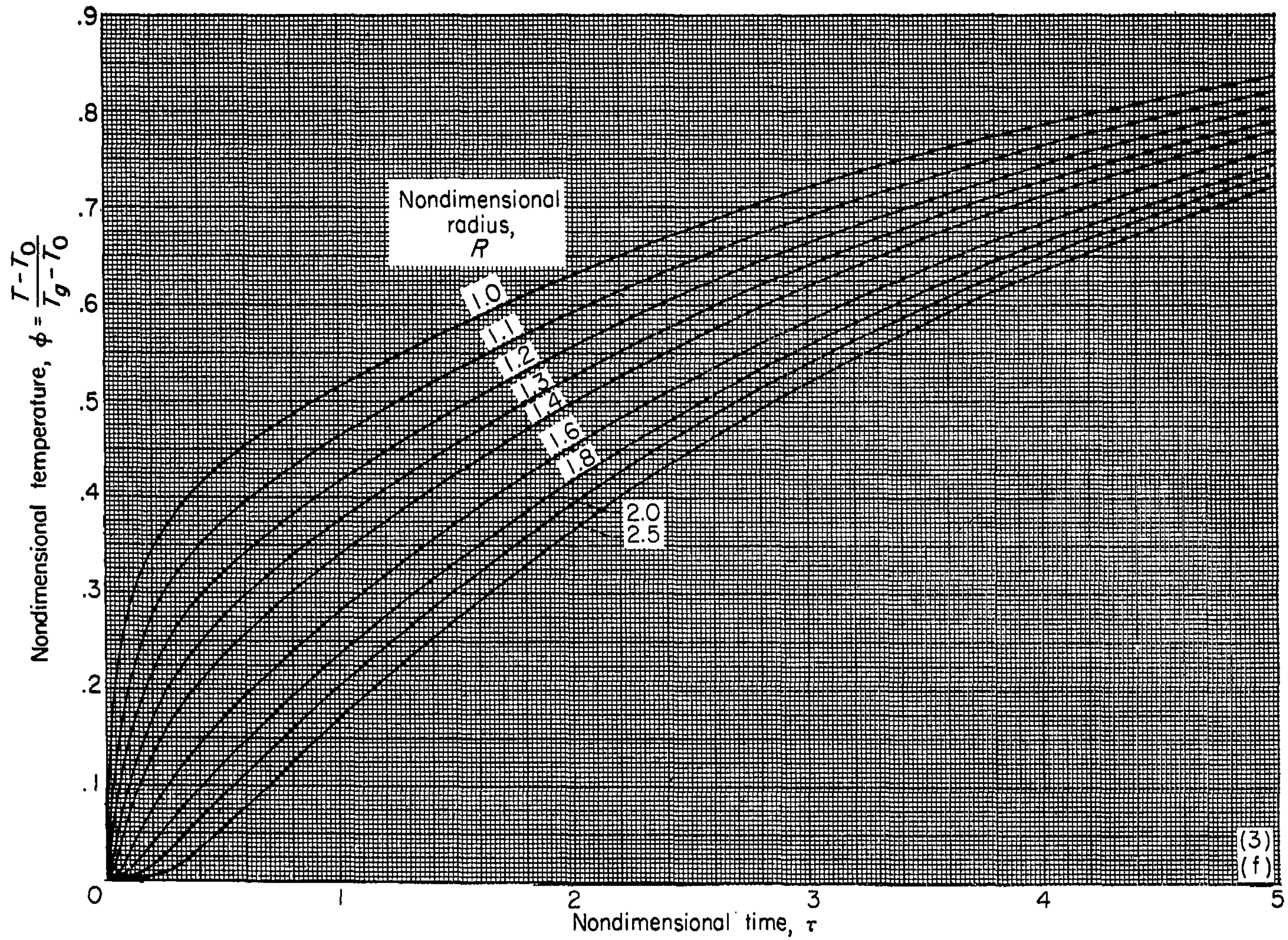
FIGURE 3.—Conitnued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(2) $H=0.5$.

(f) Continued. $R_e=2.5$.

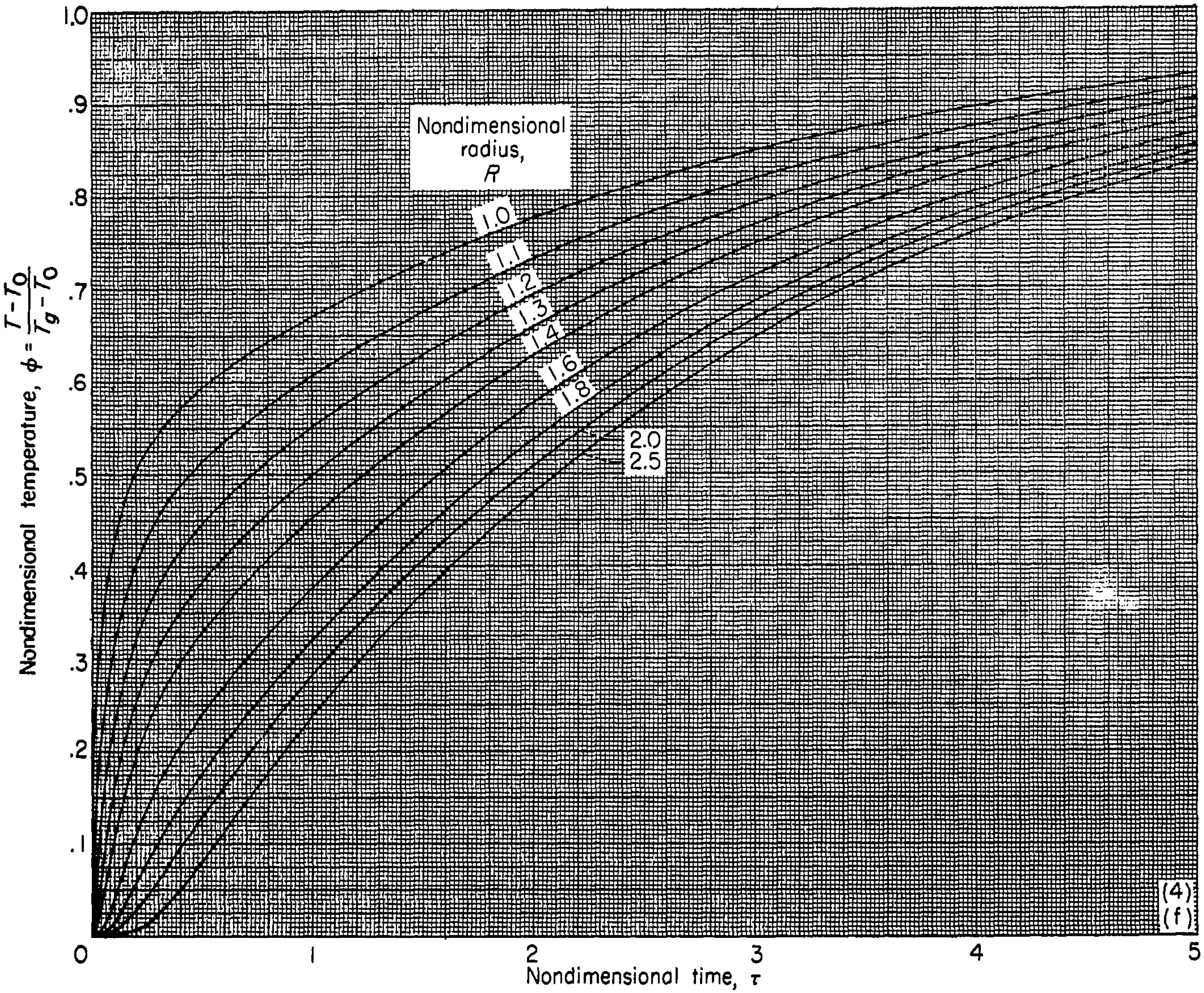
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) $H=1.1$.

(f) Continued. $R_s=2.5$.

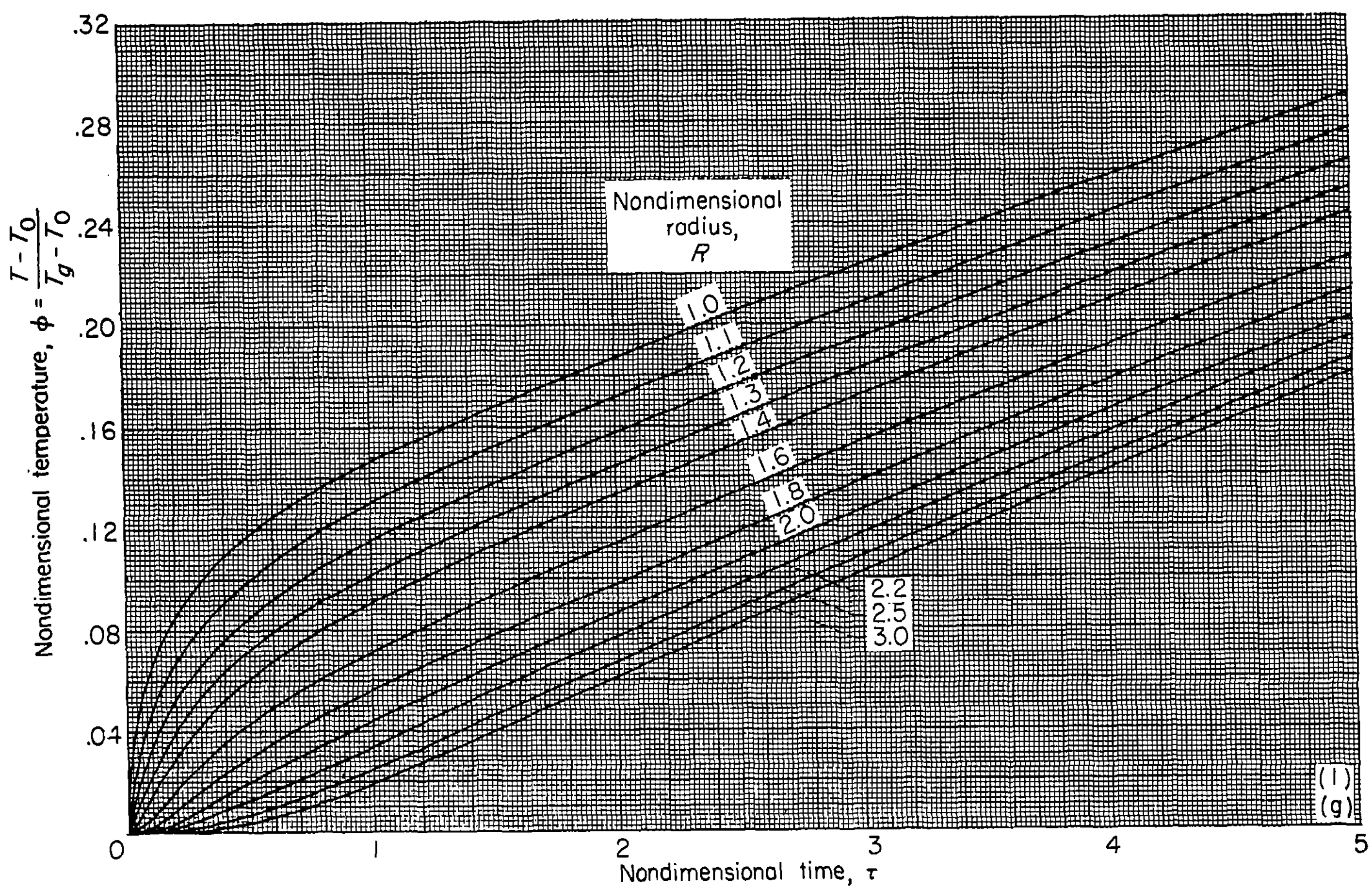
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_s and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(4) $H=2.0$.

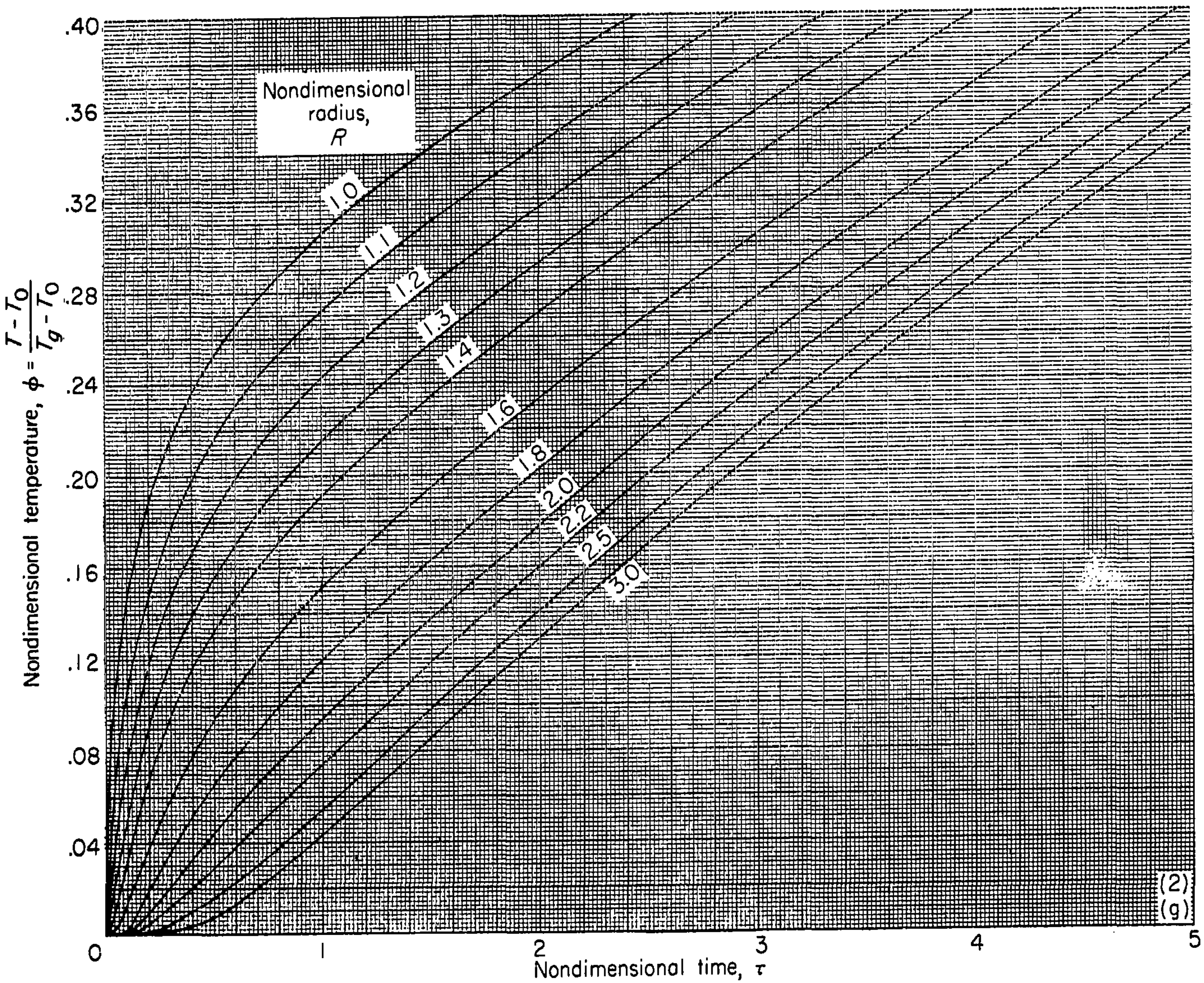
(f) Concluded. $R_e=2.5$.

Figure 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(l) $H=0.2$.
(g) $R_e=3.0$.

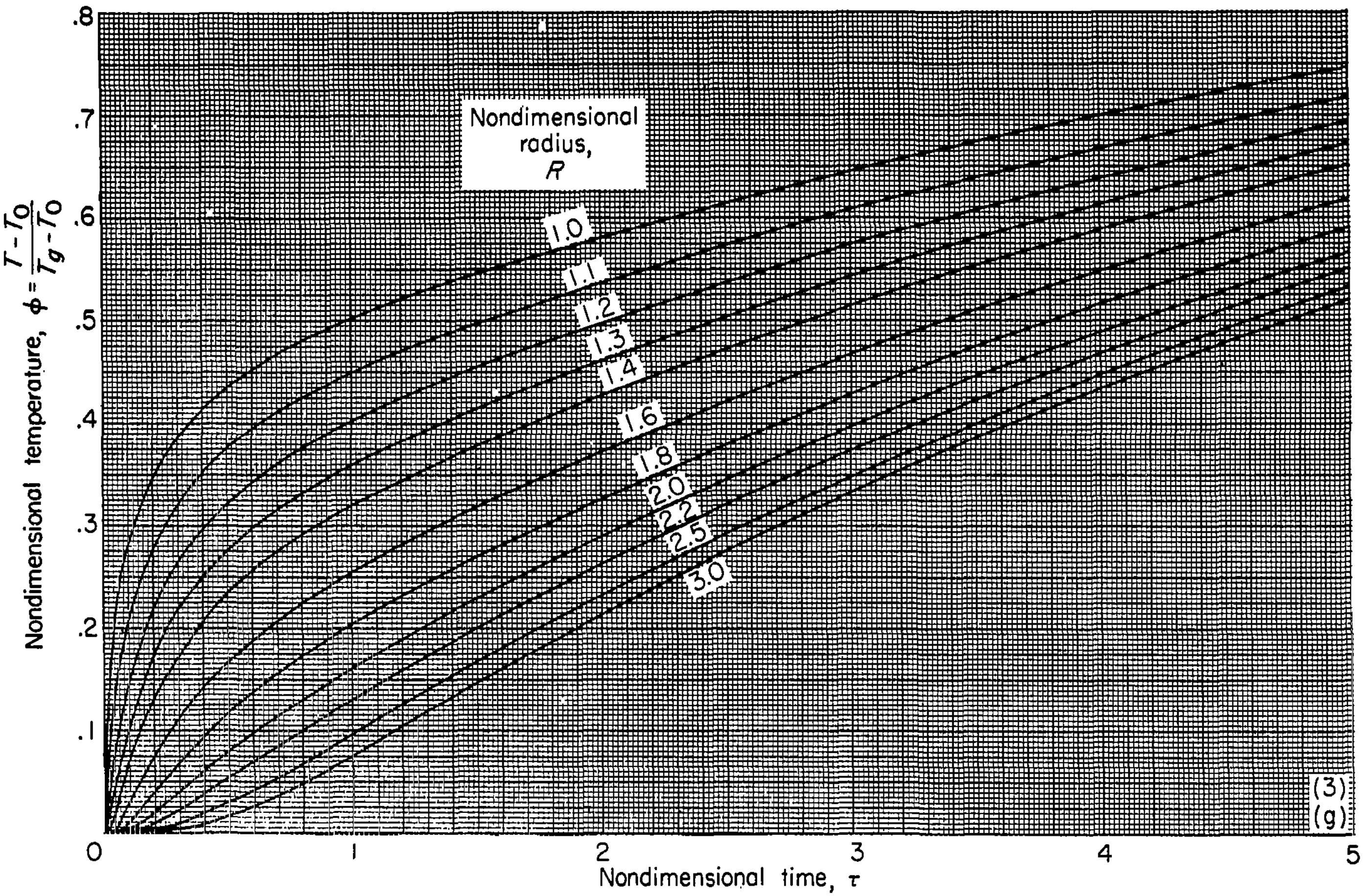
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(2) $H=0.5$.

(g) Continued. $R_e=3.0$.

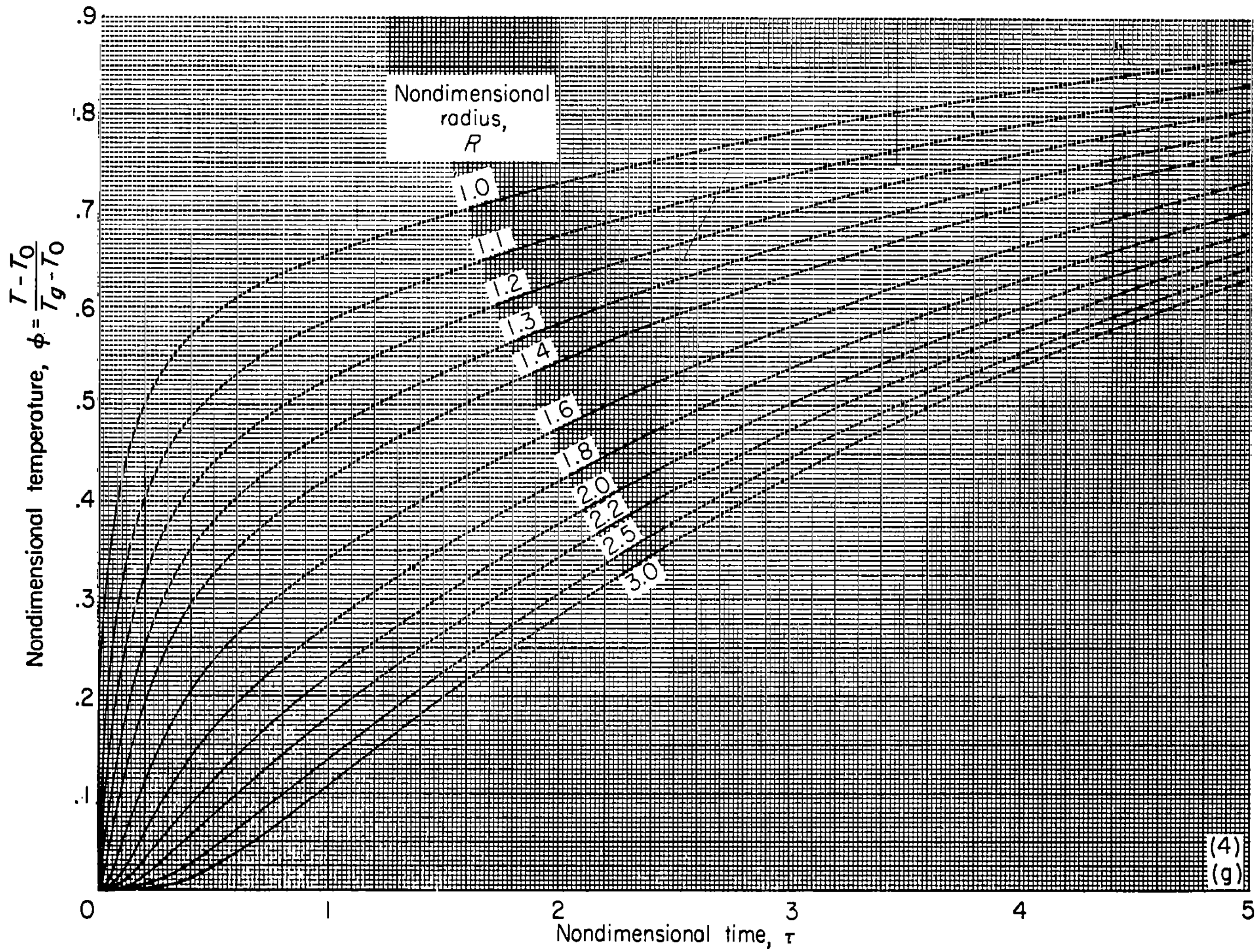
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) H=1.1.

(g) Continued. R_e=3.0.

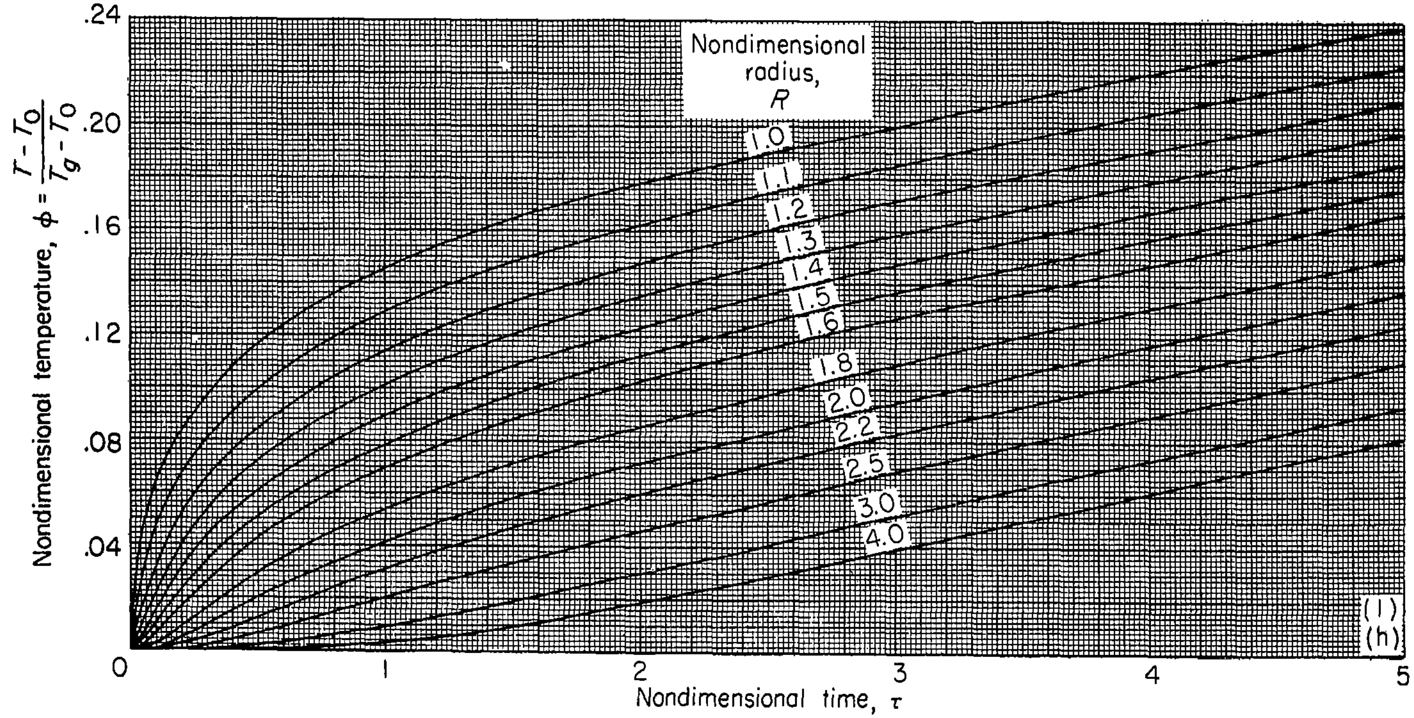
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(4) $H=2.0$.

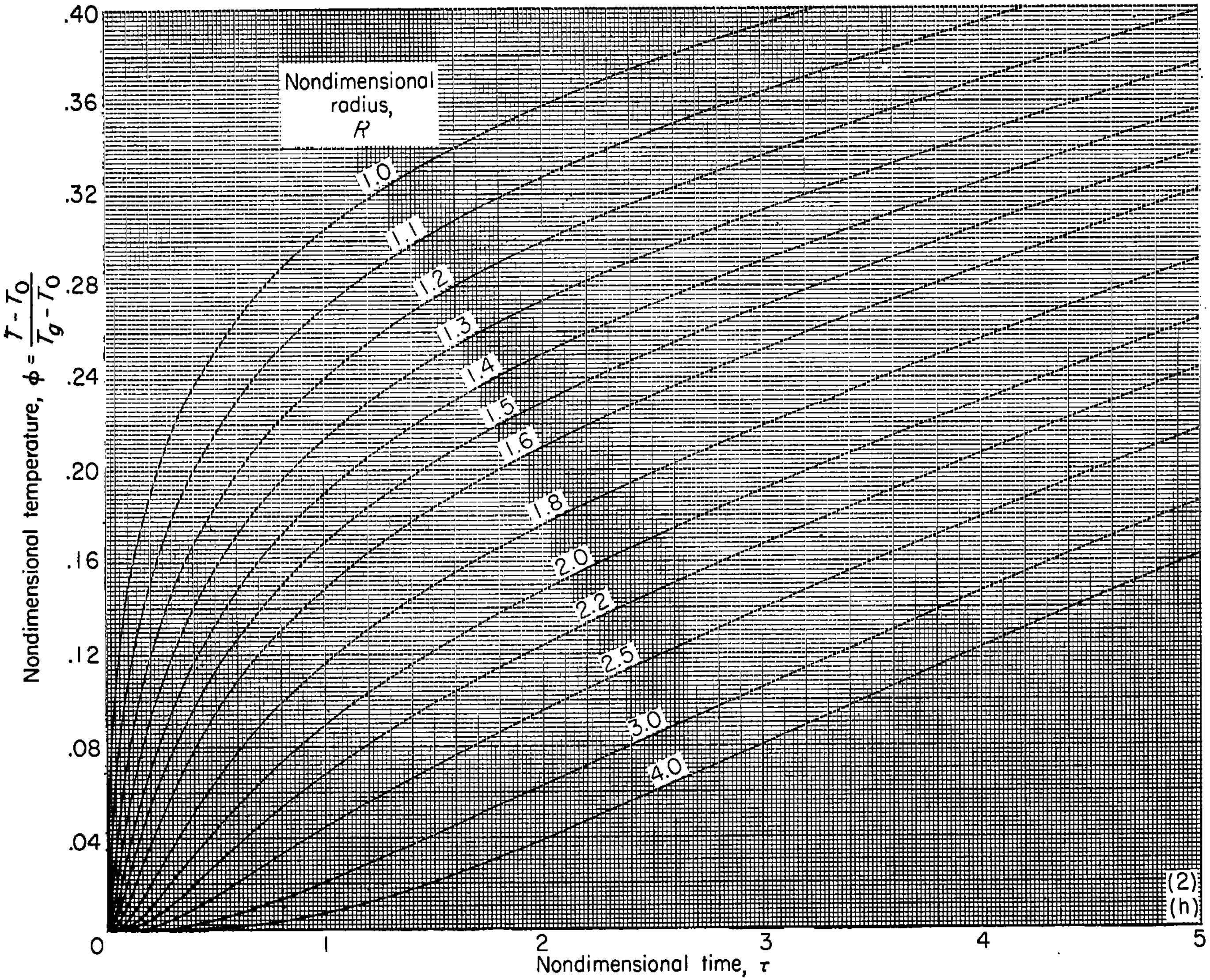
(g) Concluded. $R_s=3.0$.

Figure 3.—Continued. Time-temperature relations for various radius ratios and R_s nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(l) H=0.2.
(h) R_e=4.0.

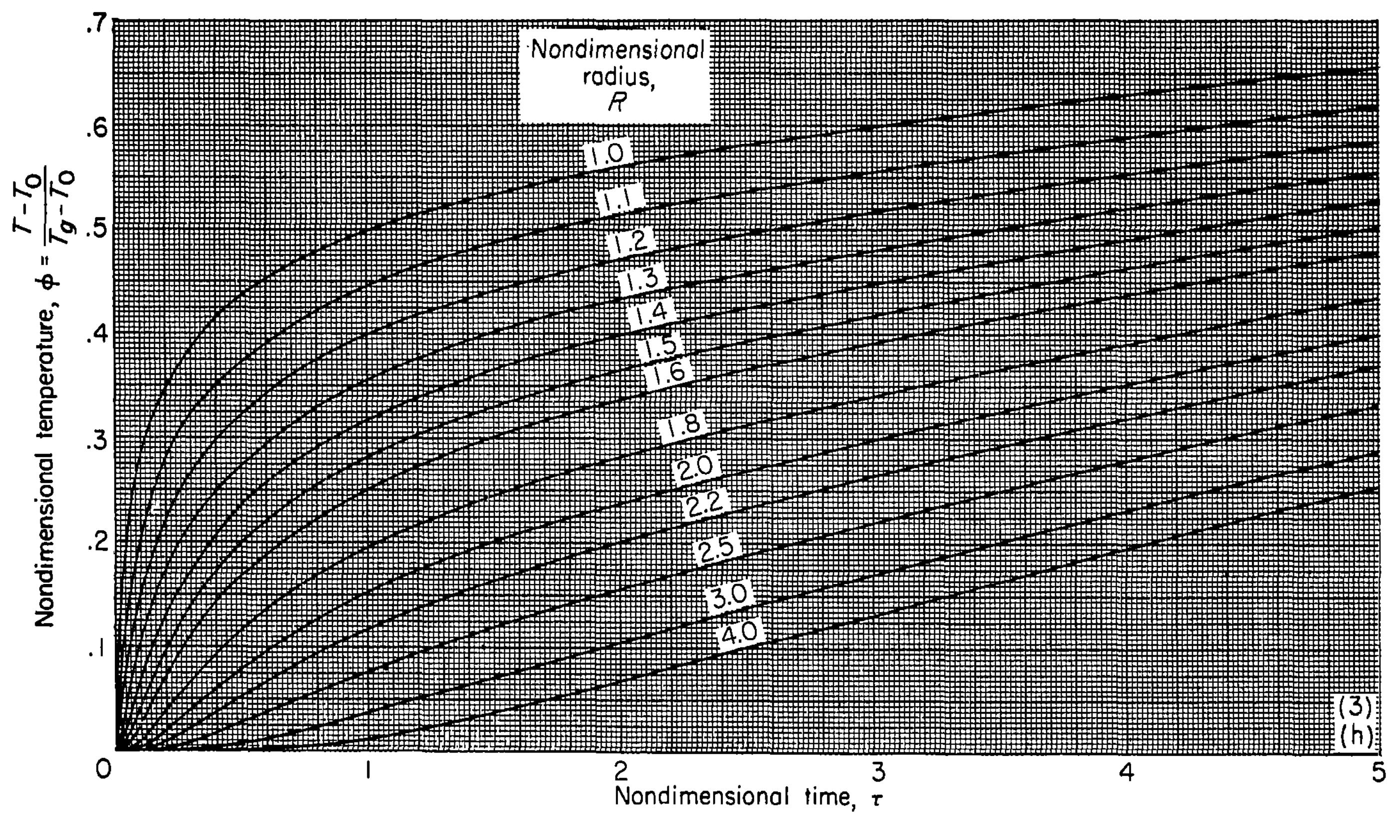
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(2) $H=0.5$.

(h) Continued. $R_e=4.0$.

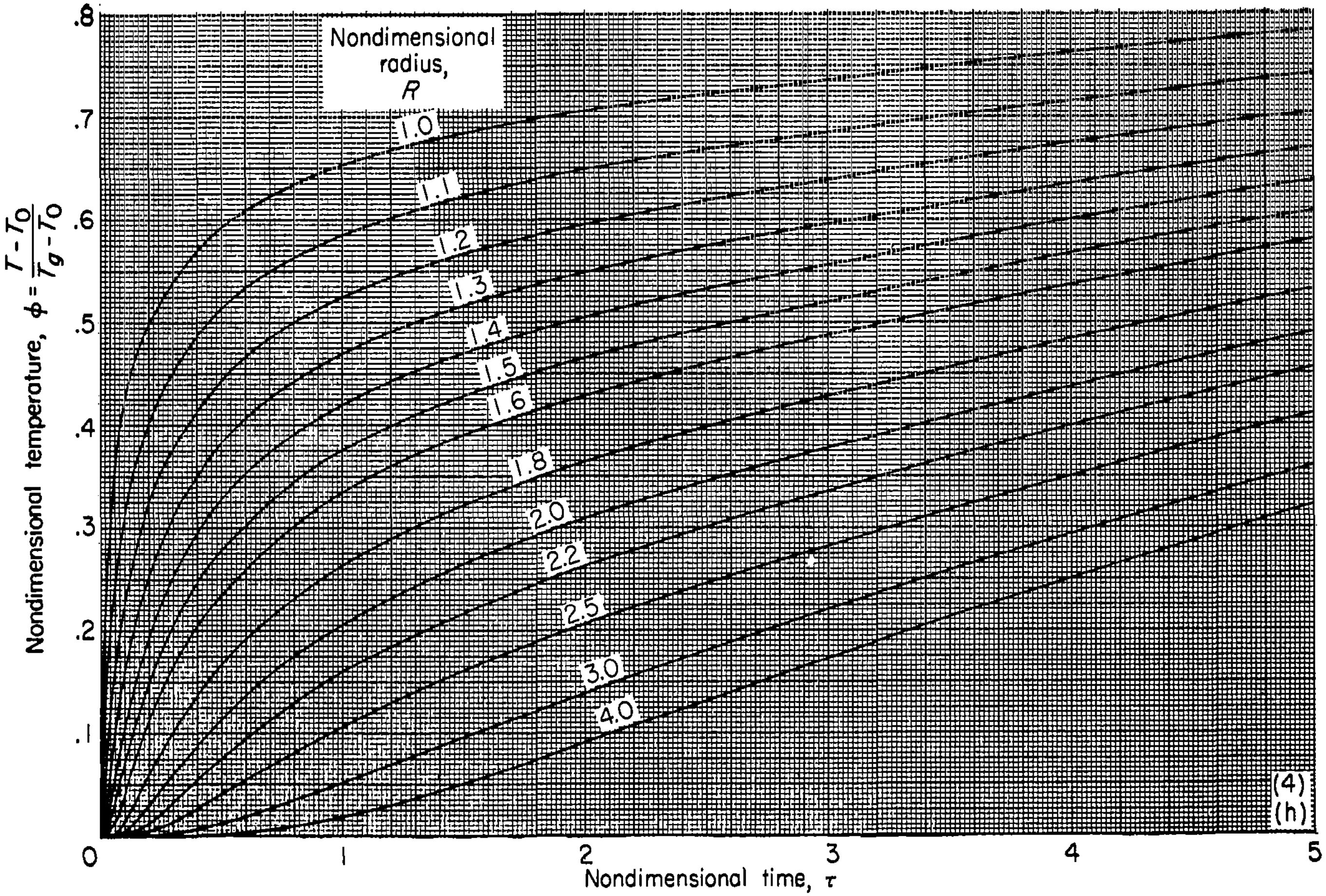
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(3) $H=1.1$.

(h) Continued. $R_e=4.0$.

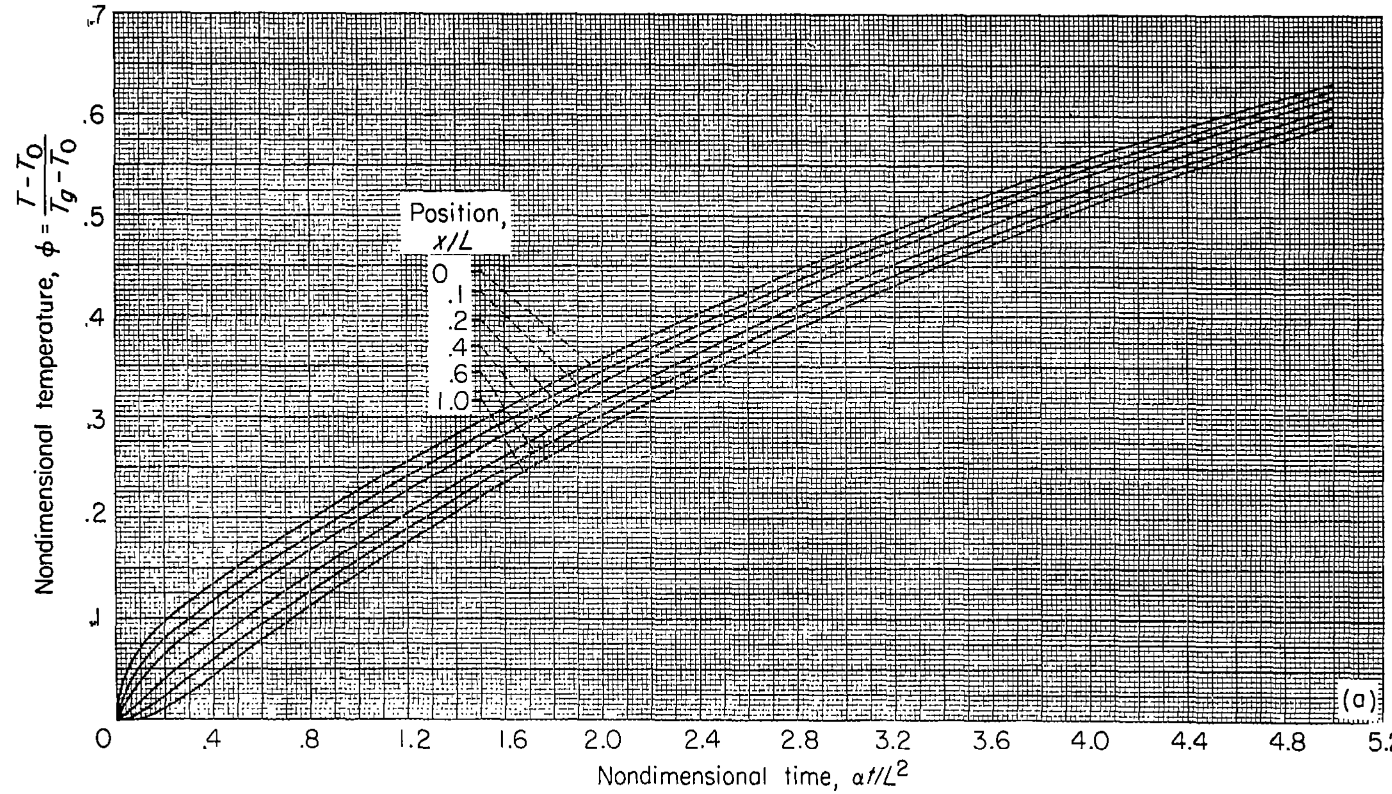
FIGURE 3.—Continued. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



(4) $H=2.0$.

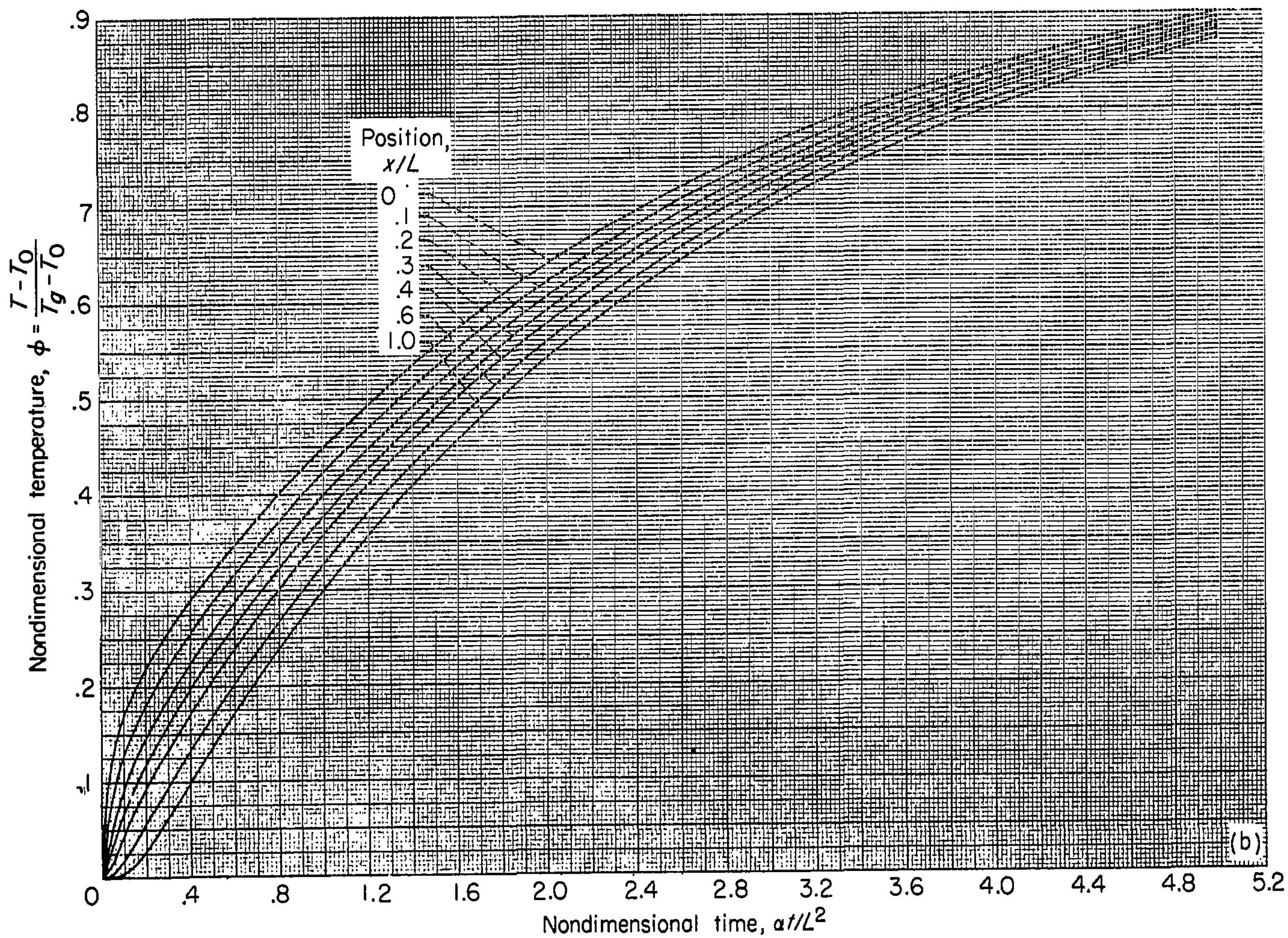
(h) Concluded. $R_e=4.0$.

FIGURE 3.—Concluded. Time-temperature relations for various radius ratios R_e and nondimensional heat-transfer coefficients H for radial heat flow in a cylinder.



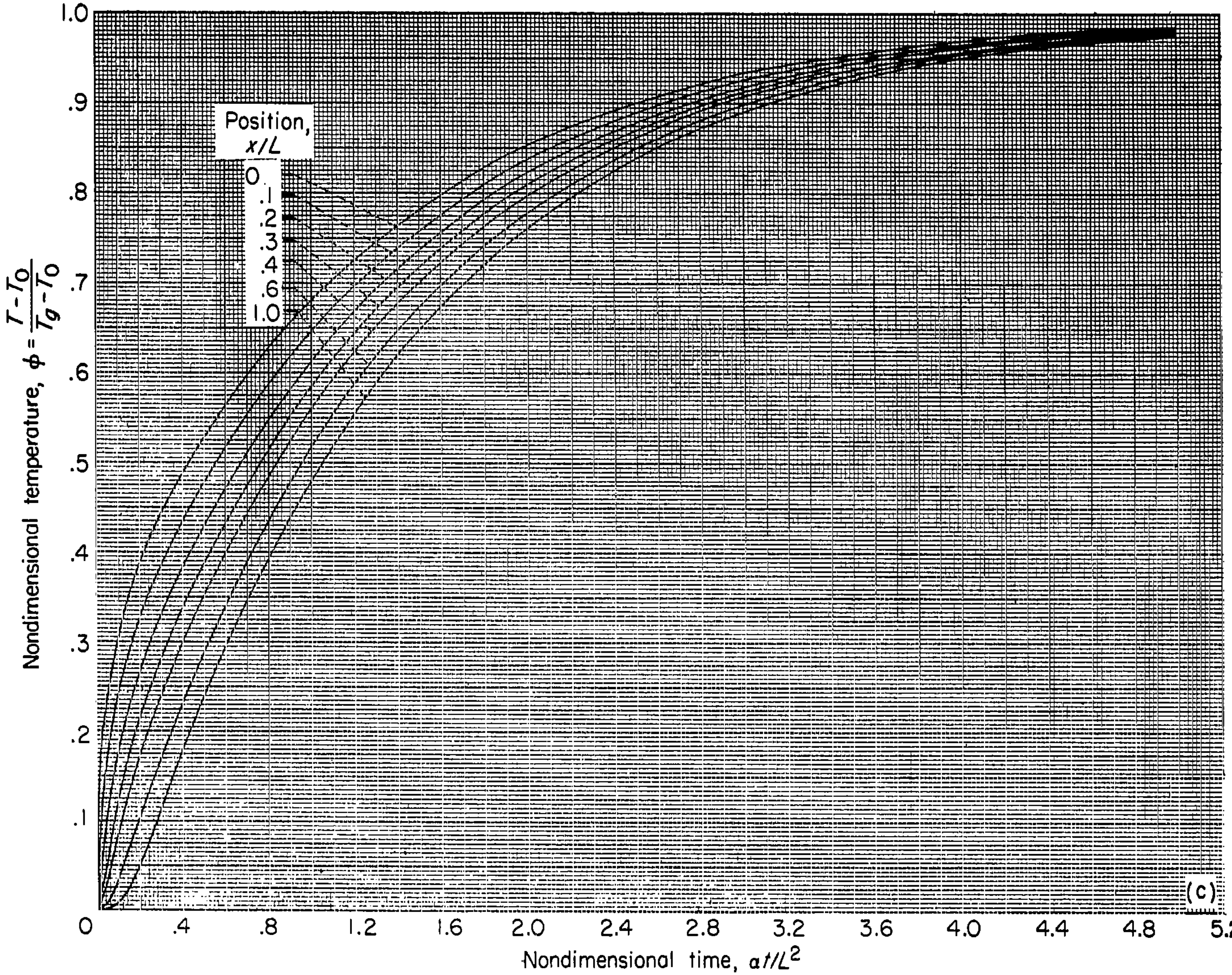
(a) $hL/k=0.2$.

FIGURE 4.—Time-temperature relations for various nondimensional heat-transfer coefficients hL/k for one-dimensional heat flow in a slab. (Larger copies of all parts of figure 4 are available from NASA, Washington 25, D.C.)



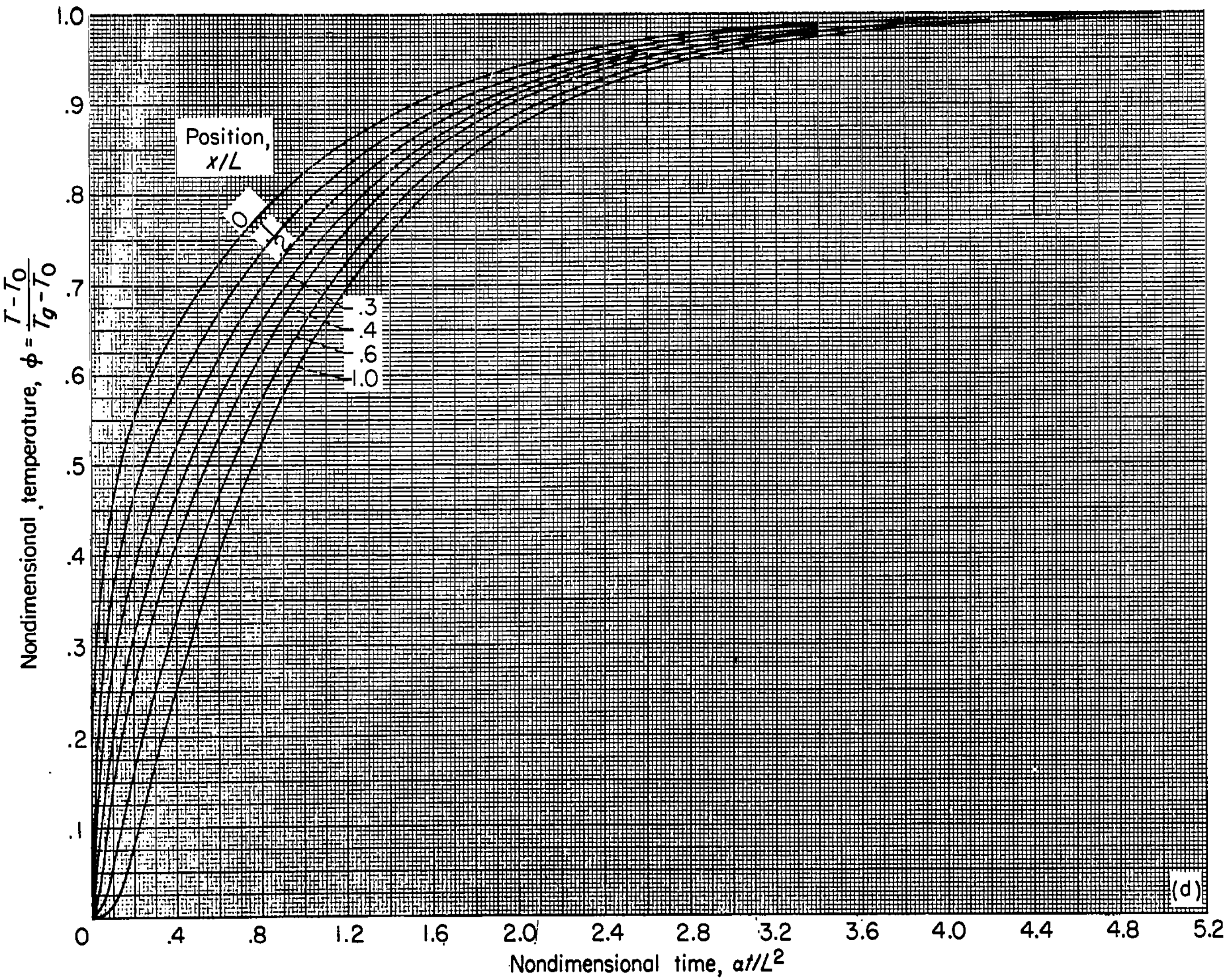
(b) $hL/k=0.5$.

FIGURE 4.—Continued. Time-temperature relations for various nondimensional heat-transfer coefficients hL/k for one-dimensional heat flow in a slab.



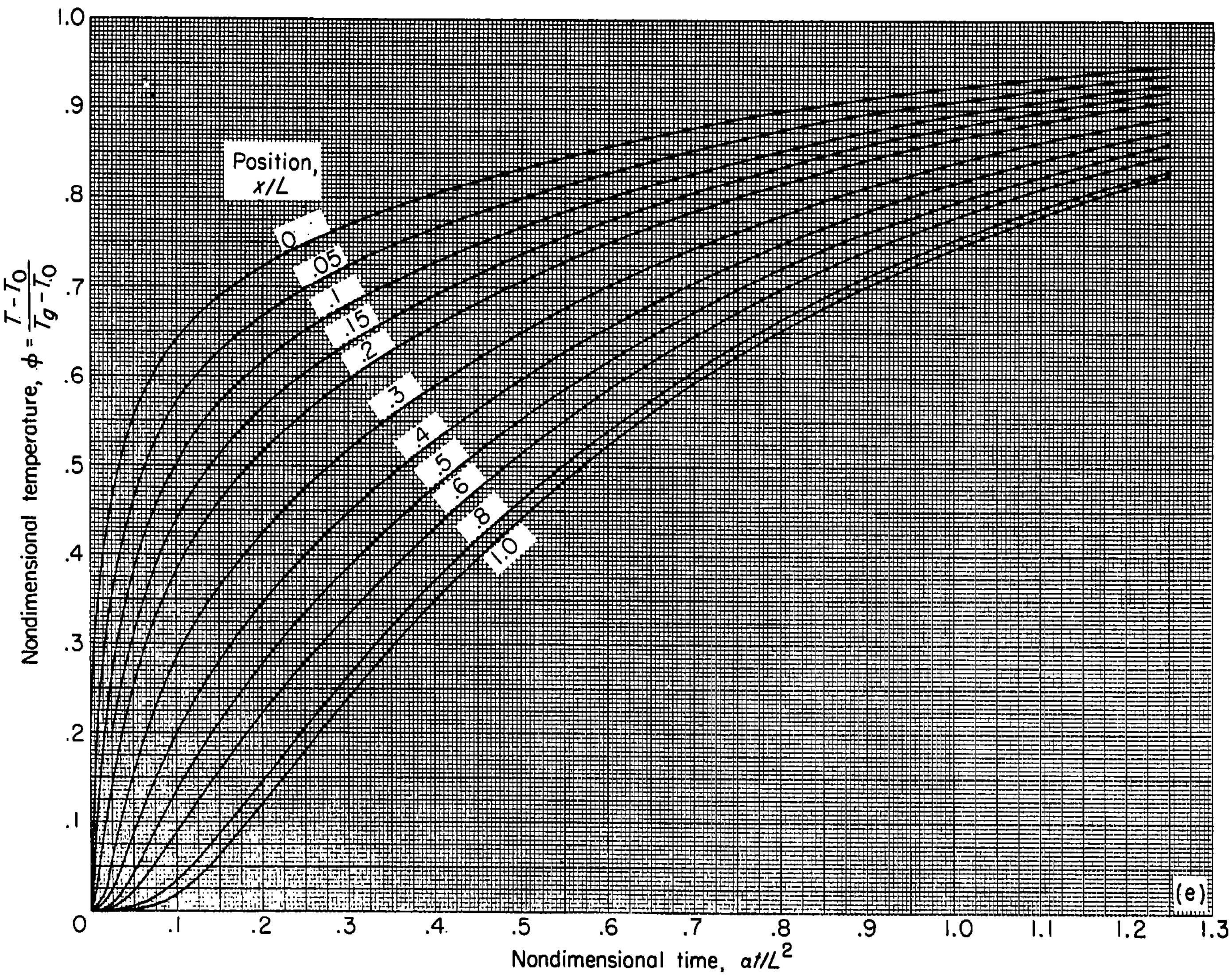
(c) $hL/k=1.1$.

Figure 4.—Continued. Time-temperature relations for various nondimensional heat-transfer coefficients hL/k for one-dimensional heat flow in a slab.



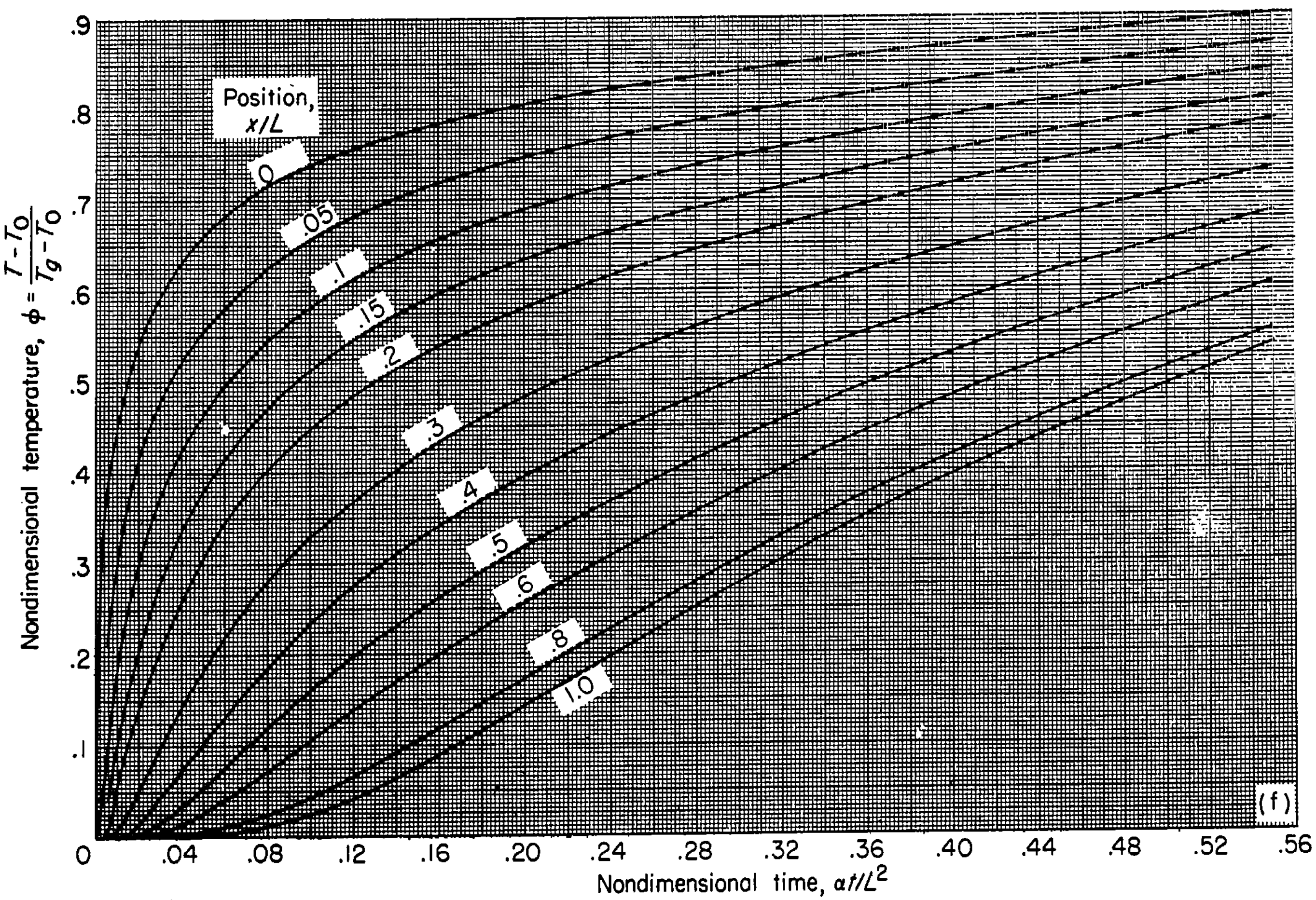
(d) $hL/k=2.0$.

FIGURE 4.—Continued. Time-temperature relations for various nondimensional heat-transfer coefficients hL/k for one-dimensional heat flow in a slab.



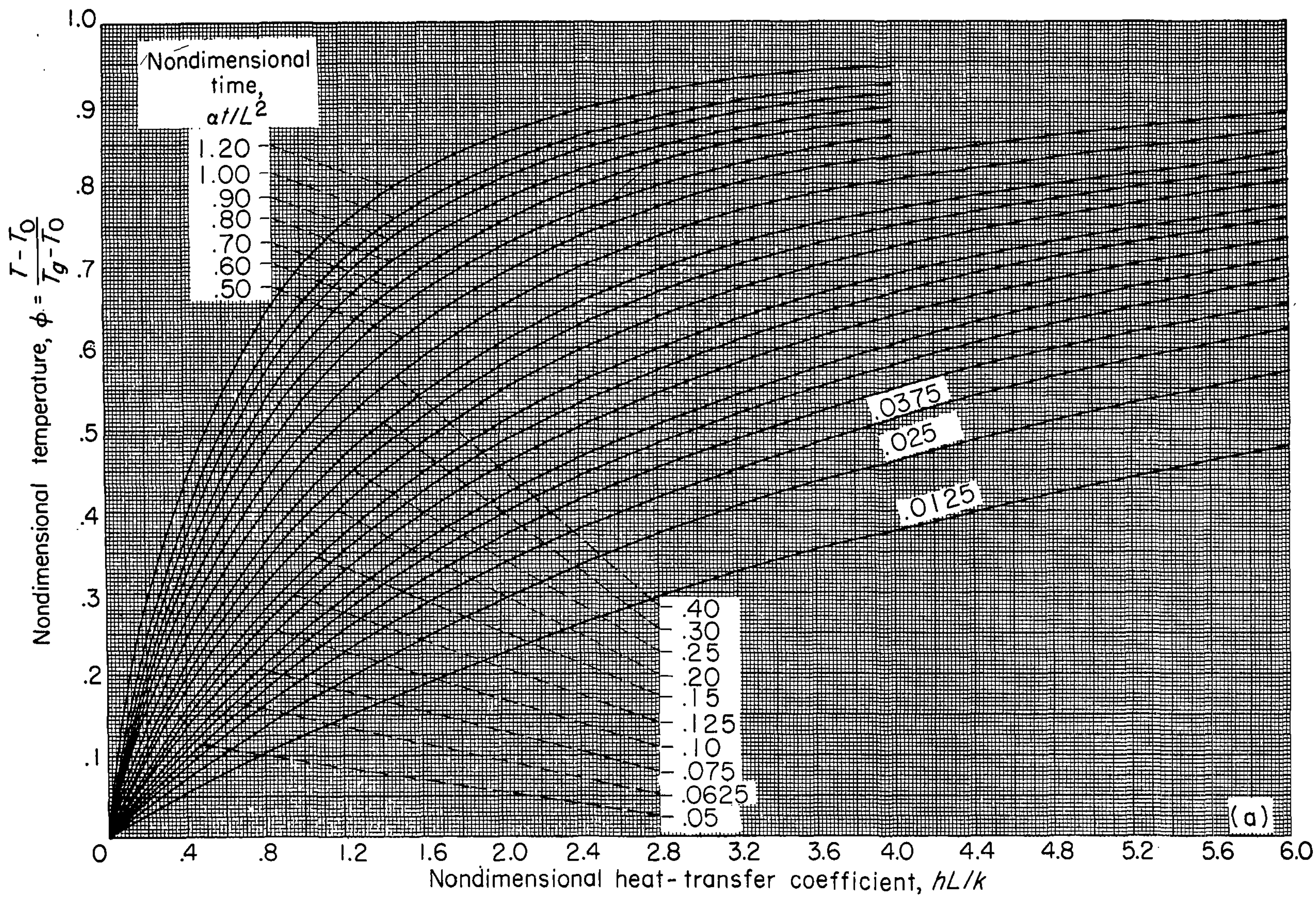
(e) $hL/k=4.0$.

Figure 4.—Continued. Time-temperature relations for various nondimensional heat-transfer coefficients hL/k for one-dimensional heat flow in a slab.



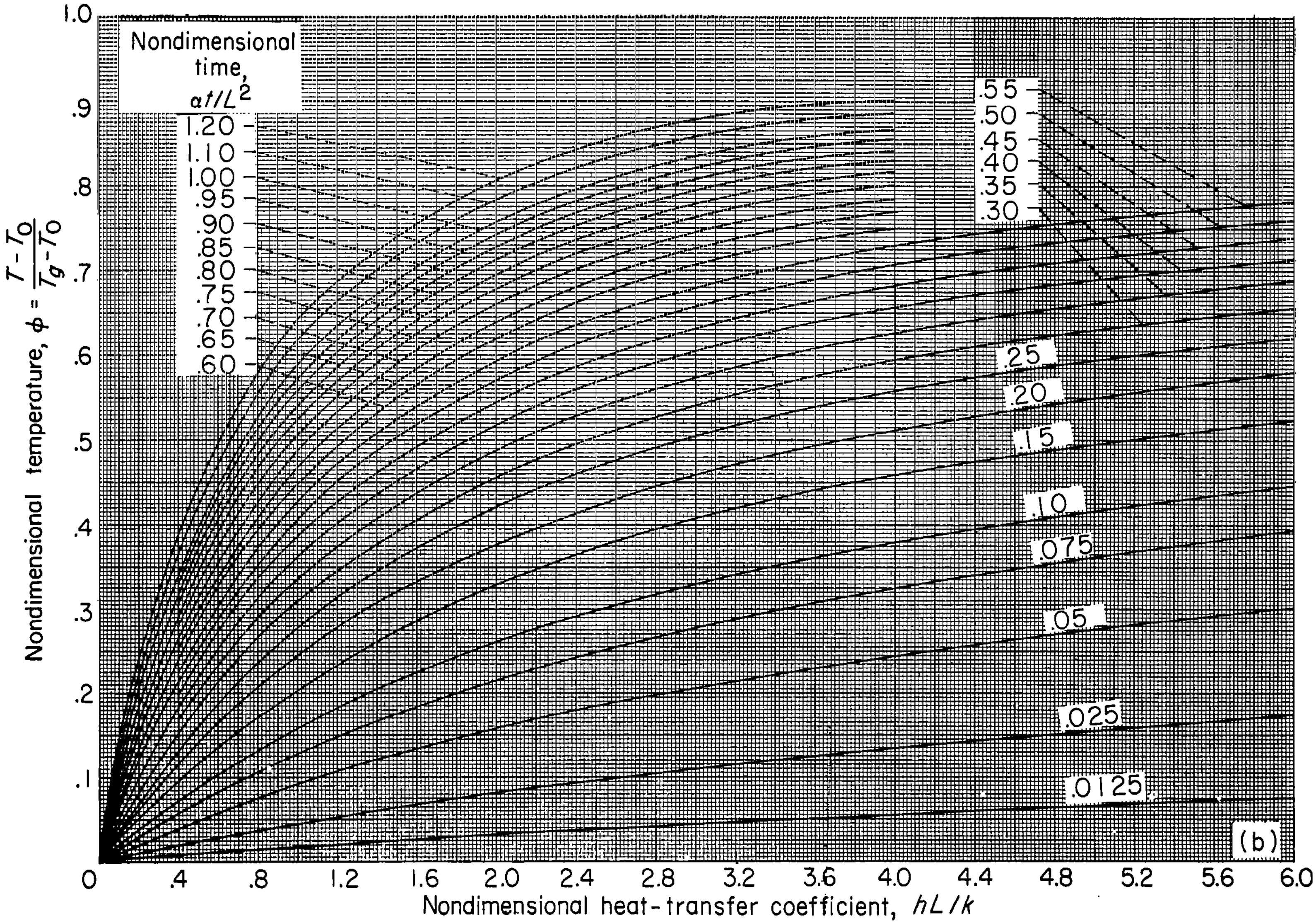
(f) $hL/k=6.0$.

FIGURE 4.—Concluded. Time-temperature relations for various nondimensional heat-transfer coefficients hL/k for one-dimensional heat flow in a slab.



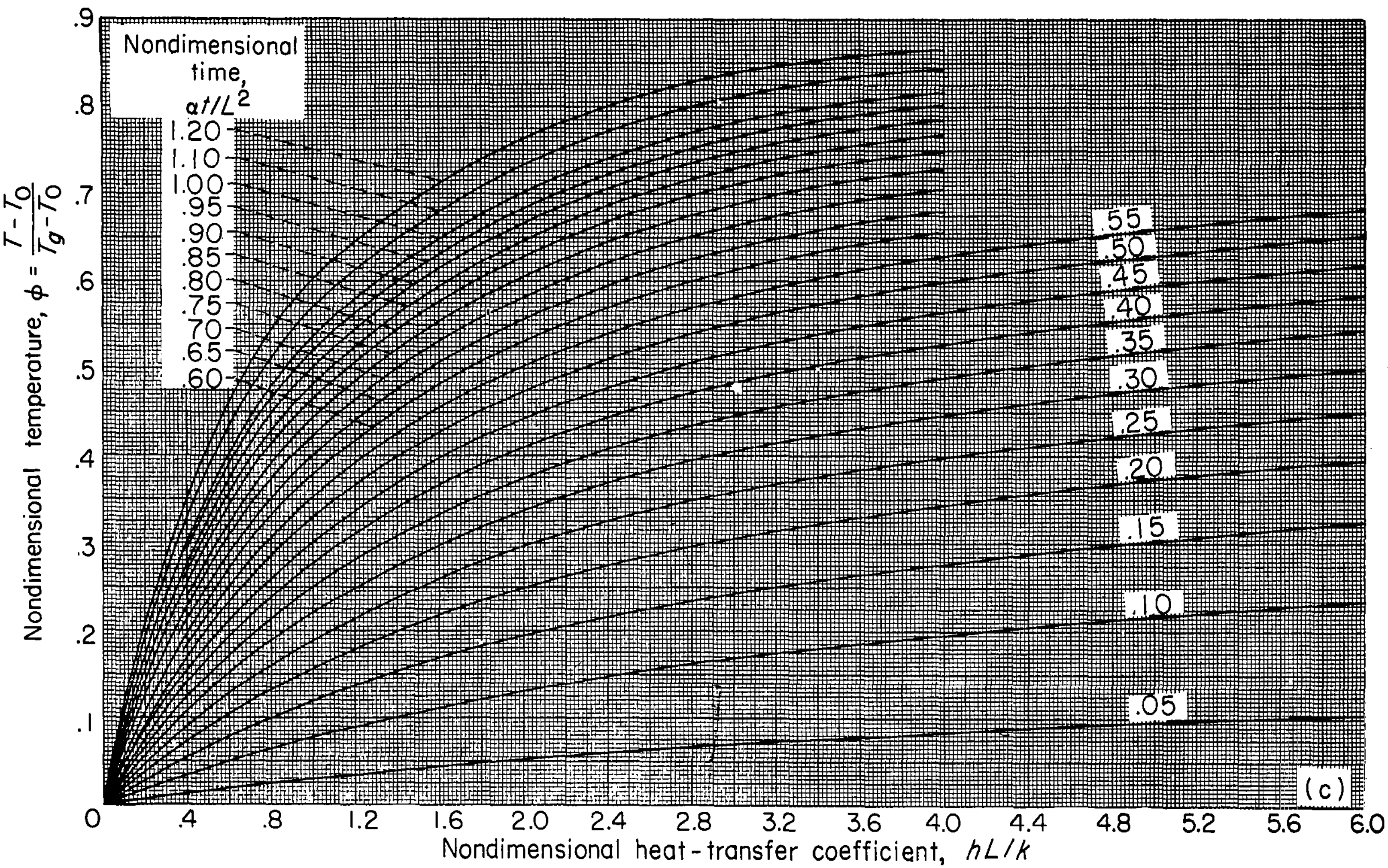
(a) $x/L=0$.

FIGURE 5.—Temperature—heat-transfer-coefficient relations for various positions x/L in a slab with time $\alpha t/L^2$ as a parameter. (Larger copies of all parts of figure 5 are available from NASA, Washington 25, D.C.)



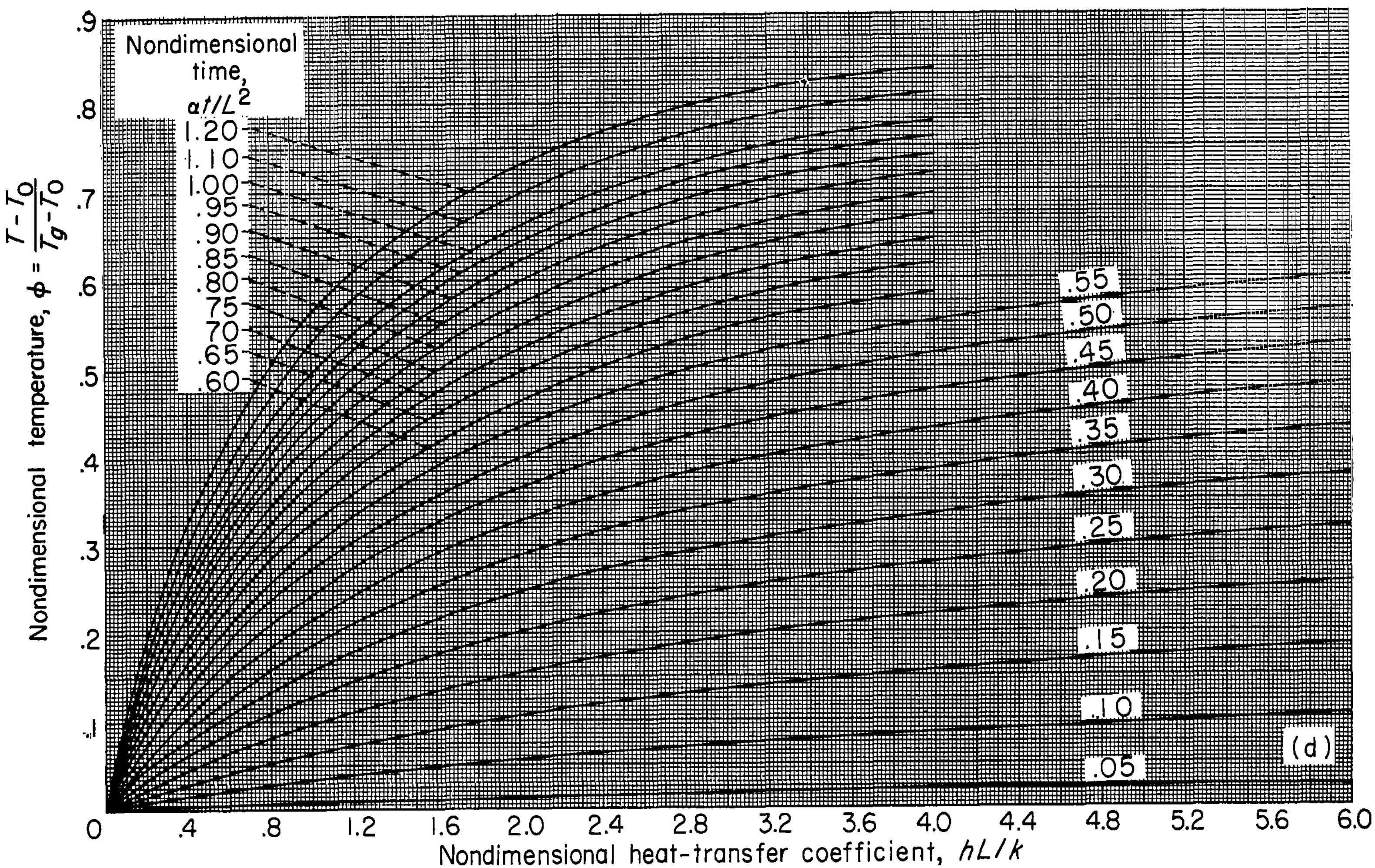
(b) $x/L=0.2$.

FIGURE 5.—Continued. Temperature—heat-transfer-coefficient relations for various positions x/L in a slab with time $\alpha t/L^2$ as a parameter.



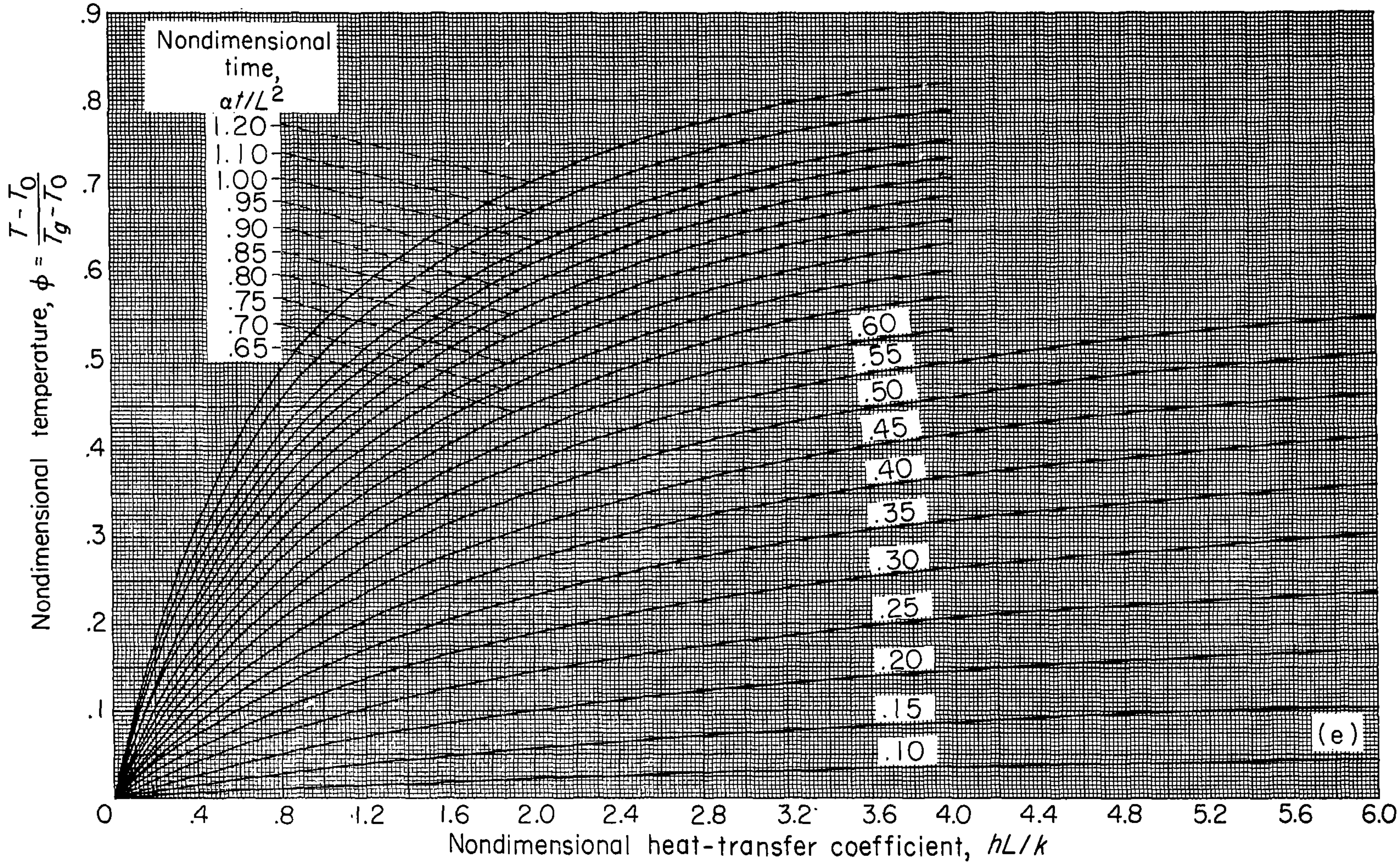
(c) $x/L=0.4$.

FIGURE 5.—Continued. Temperature—heat-transfer-coefficient relations for various positions x/L in a slab with time $\alpha t/L^2$ as a parameter.



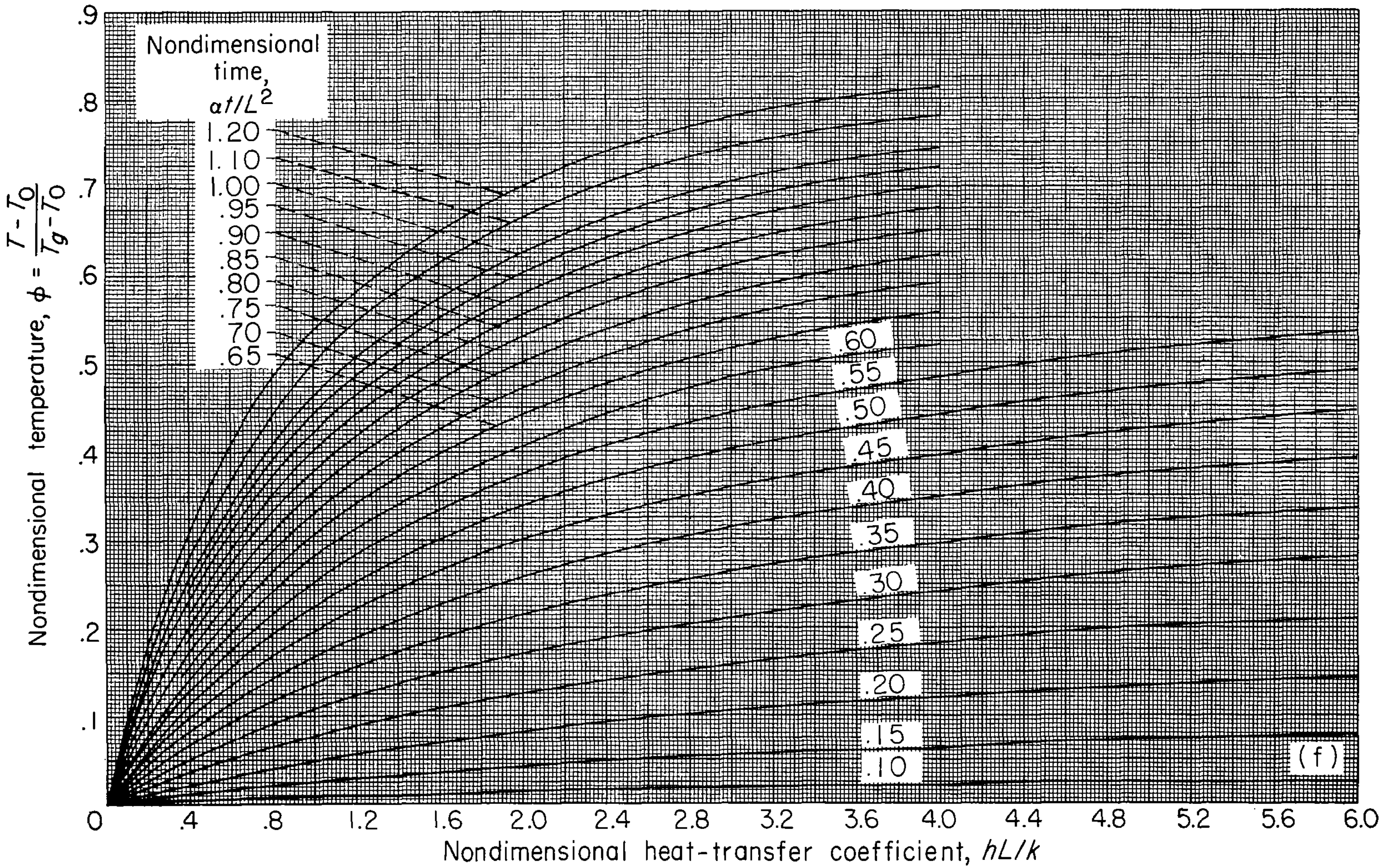
(d) $x/L=0.6$.

FIGURE 5.—Continued. Temperature—heat-transfer-coefficient relations for various positions x/L in a slab with time $\alpha t/L^2$ as a parameter.



(e) $x/L=0.8$.

FIGURE 5.—Continued. Temperature—heat-transfer-coefficient relations for various positions x/L in a slab with time $\alpha t/L^2$ as a parameter.



(f) $x/L=1.0$.

FIGURE 5.—Concluded. Temperature—heat-transfer-coefficient relations for various positions x/L in a slab with time $\alpha t/L^2$ as a parameter.

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